7

Test

We use an electricity bill as a hook on which to hang an introduction to functions in general and linear functions in particular, in algebra and in spreadsheets. Then we apply what we've learned to study taxes — sales, income and Social Security. You'll also find here a general discussion of energy and power.

Chapter goals:

Goal 7.1. Understand direct proportion as a linear equation with intercept 0.

Goal 7.2. Study situations governed by linear equations.

Goal 7.3. Stress the meaning and units of the slope and intercept.

Goal 7.4. View functions as tables, graphs and formulas.

Goal 7.5. Construct flexible spreadsheets to model linear equations.

Goal 7.6. Understand the piecewise linear income tax computations.

Goal 7.7. Sort out the confusing distinction between energy and power.

7.1 Rates

We began Chapter **??** with a discussion of the relationship

$$\text{distance} = \text{rate} \times \text{time},$$

or, with units,

$$\text{distance (miles)} = \text{rate (miles/hour)} \times \text{time (hours)}. \tag{7.1}$$

There we worked with particular numbers; now we want to look at that relationship a little more generally. If the rate is fixed (say 60 miles/hour), then we can find the distance traveled whenever we know the time. To say that with some algebra, write D for distance and T for time. Then

$$D = 60 \times T,$$

or, more generally,

$$D = r \times T,$$

where r is the rate of travel, in miles/hour.

That formula says that distance traveled is *proportional* to travel time. The rate in miles per hour is the *proportionality constant*. If you drive for twice as long you go twice as far. If you drive ten times as long you go ten times as far.

In Section **??** we introduced the units (gallons per hundred miles) as a useful way to measure automobile fuel economy: the amount of gas you use is proportional to the distance you drive. Let F be the amount of gasoline you use, in gallons, to drive D hundred miles. Then

$$F = r \times D$$

where the proportionality constant r is the fuel use rate, with units gallons per hundred miles. If you drive 10 times as far you use 10 times as much gas. You don't use any gas at all just sitting in the driveway (unless you're idling to warm up the car).

The proportionality constant is always a rate: it appears with units. In these examples the units are miles per hour and gallons per 100 miles.

The unit pricing discussion in Section **??** provides more examples of proportionality.

Finally, sales tax is computed as a proportion. If you spend twice as much you pay twice as much tax. The tax rate is the proportionality constant. If it's 5% then you pay (five dollars of tax) per (hundred dollars of purchase).

7.2 Reading your electricity bill

The more electricity you use at home, the more you pay. But the relationship isn't quite proportional. You don't pay twice as much to use twice as much. Figure 7.1 shows a simple sample electricity bill based on one we found on a British website. [R1]

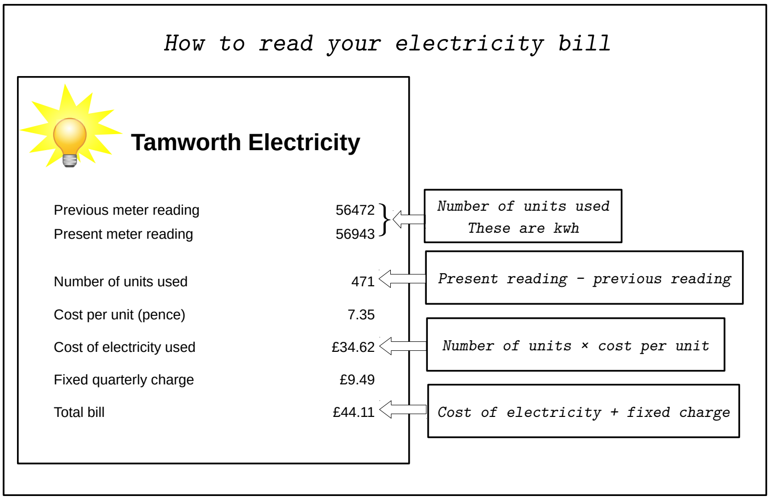


Figure 7.1. Tamworth electricity bill

This bill explains itself. We'll study it before we look at a real one. Since it comes from Great Britain the costs are expressed in pounds and pence rather than dollars and cents, and it comes once a quarter (every three months) rather than once a month, but you can ignore that while you read it — from the bottom up.

The last line is the total bill, computed as

$$\boxed{\texttt{Cost of electricity + fixed charge}}\,.$$

Checking the arithmetic:

$$£34.62 + £9.49 = £44.11.$$

The third line from the bottom explains the £34.62:

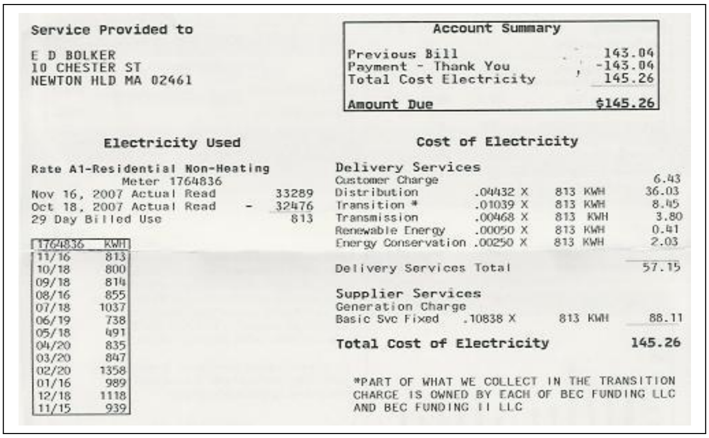
$$\boxed{\texttt{Number of units} \times \texttt{cost per unit}}\,.$$

That's our old friend proportionality. The previous line gives the proportionality constant: the cost per unit as 7.35 pence per unit. Later on in the document we're told that a unit is just a kilowatt-hour, abbreviated "kwh". (It's too bad the bill talks about "units" instead of just "kwh" since for us "units" has a more general meaning.)

There are 100 pence in a pound, so the cost per unit is

$$0.0735 \frac{£}{\text{kwh}}.$$

(The computation would have been much more complicated before February 15, 1971 — the day England converted from pounds/shillings/pence to decimal currency. See `news.bbc.co.uk/onthisday/hi/dates/stories/february/15/newsid_2543000/2543665.stm` .)



Service Provided to

E D BOLKER
10 CHESTER ST
NEWTON HLD MA 02461

Account Summary

Previous Bill	143.04
Payment - Thank You	-143.04
Total Cost Electricity	145.26
Amount Due	$145.26

Electricity Used

Rate A1-Residential Non-Heating
Meter 1764836

Nov 16, 2007 Actual Read	33289
Oct 18, 2007 Actual Read -	32476
29 Day Billed Use	813

1764836	KWH
11/16	813
10/18	800
09/18	814
08/16	855
07/18	1037
06/19	738
05/18	491
04/20	835
03/20	847
02/20	1358
01/16	989
12/18	1118
11/15	939

Cost of Electricity

Delivery Services			
Customer Charge			6.43
Distribution	.04432 X	813 KWH	36.03
Transition *	.01039 X	813 KWH	8.45
Transmission	.00468 X	813 KWH	3.80
Renewable Energy	.00050 X	813 KWH	0.41
Energy Conservation	.00250 X	813 KWH	2.03
Delivery Services Total			57.15
Supplier Services			
Generation Charge			
Basic Svc Fixed	.10838 X	813 KWH	88.11
Total Cost of Electricity			**145.26**

*PART OF WHAT WE COLLECT IN THE TRANSITION CHARGE IS OWNED BY EACH OF BEC FUNDING LLC AND BEC FUNDING II LLC

Figure 7.2. NStar electricity bill (2007)

In the current quarter this customer used 471 kwh of electricity — the difference between the meter reading before and after the quarter.

Here's all the arithmetic, with units:

$$£44.11 = 0.0735 \frac{£}{\cancel{\text{kwh}}} \times 471 \, \cancel{\text{kwh}} + £9.49.$$

That English bill is easy to read. Figure 7.2 shows a real one that's a little more complex, from NStar, in Boston.

We can identify the same two components. The fixed charge is the $6.43 labelled "Customer Charge." It's the part of the $145.26 total that does not depend on the amount of electricity used — in this case, 813 kwh. The six lines on the bill that do depend on that contribute

$$\begin{aligned} (0.04432 + 0.01039 + 0.00468 & \\ + 0.00050 + 0.00250 + 0.10838) & \times 813 \\ &= 0.17077 \times 813 \\ &= 138.83601 \end{aligned}$$

to the total bill, which is

$$\$145.26 = 0.17077 \frac{\$}{\cancel{\text{kwh}}} \times 813 \, \cancel{\text{kwh}} + \$6.43.$$

Note that NStar rounded $138.83601 down to $138.83 rather than up to the nearest penny. We should be grateful for small favors.

7.3 Linear functions

So far *Common Sense Mathematics* has called for hardly any algebra. Now a little bit will come in handy.

Suppose you buy your electricity from NStar as in the example above and want to study how your bill changes when you use different amounts of electricity. The monthly \$6.43 Customer Charge does not change. The rest of your bill is proportional to the amount of electricity. The proportionality constant is 0.17077 \$/kwh in the sample bill. We will assume that it does not change, although in fact it does change slightly when the electric company changes its rates.

If in a given month you use E kwh of electricity your total bill B can be computed with the formula

$$B\$ = 0.17077 \frac{\$}{\cancel{\text{kwh}}} \times E \ \cancel{\text{kwh}} + \$6.43.$$

That formula captures how the dollar amount of your electricity bill depends on the amount of electricity you use, measured in kwh. The first term is the part that's proportional to the amount of electricity used. The second term (the amount \$6.43) is fixed. It represents the electric company's fixed costs: things like generating the bill and mailing it to you and maintaining the power lines on the street in front of your house. Those are expenses they must cover even if you're on vacation and have turned off all the appliances.

You probably encountered a similar formula once in an algebra class — it may look more familiar without the units

$$B = 0.17077 \times E + 6.43.$$

It may look even more familiar if we call the variables by the traditional names x (for the independent variable) and y (for the dependent) instead of E and B:

$$y = 0.17077x + 6.43.$$

This is a *linear function*, which standard algebra texts write in *slope-intercept* form

$$y = mx + b.$$

In this example the *slope* m is $0.17077 \frac{\$}{\text{kwh}}$ and the *intercept* b is \$6.43. For the English bill the slope is $0.0735 \frac{£}{\text{kwh}}$ and the intercept is £9.49.

There are many everyday examples where a linear equation describes how a total is computed by adding a fixed amount to a varying part that's a proportion. The fixed amount is the intercept. The proportionality constant is the slope.

- The most familiar examples are the ones where the intercept is 0: all the ones in Section 7.1.
- When renting a truck, the amount you pay is

  (rate in dollars/mile) × (miles driven) + (fixed charge) .

- Your monthly cell phone bill might be

  (rate in dollars per minute) × (number of minutes) + (fixed fee) .

  (A real cell phone bill will probably be more complicated, perhaps with separate charges for phone minutes, text messages and data transfer, perhaps with some of each kind of use built in to the fixed fee.)

- If you work as a salesperson and your commission is 15% of total sales your total wages are

$$0.15 \times (\text{total sales}) + (\text{your base salary})\ .$$

The pattern is

$$\text{total} = (\text{rate}) \times (\text{amount of some quantity}) + (\text{fixed constant}).$$

In each case the slope is the rate and the intercept is the fixed constant. The units of a slope are always those of a rate. In the truck example, the slope is the rate in dollars per mile; in the cell phone example, the units of the slope are dollars per minute; in the salesperson example, the slope is 0.15 dollars of commission per dollar of total sales.

Think of the intercept as an initial or starting value; it's what happens when the input is zero. It has proper units too — in each of these examples that unit is dollars. If you rent a truck but don't drive it anywhere, you still pay the fixed charge. If you make no cell phone calls you still pay the fixed fee. If you don't sell anything in a month, your commission is $0 but you still earn your base salary.

7.4 Linear functions in a spreadsheet

In Section 7.2 we saw that the amount you pay for electricity in a month is a linear function of the amount you use. In this section we'll use Excel to calculate electricity bills and to draw a picture of the results.

Figure 7.3 is a screen shot of the Excel spreadsheet `TamworthElectric.xlsx`. We put the slope 0.735 in cell `C4`, with its units £/kwh in cell `D4`. We put the intercept 9.49 in cell `C5` and the units (£) in cell `D5`.

Then we entered column labels in cells `B7:C8` and a few values in rows `9` through `13` in column `B`. Finally, we asked Excel to calculate the electricity bills in column `C`. To do that, we started with the formula

```
=C4*B9+C5
```

in cell `C9`. The = sign tells Excel to multiply the numbers in cells `C4` and `B9` and add the number in `C5`. The result is `9.49`, as expected.

The next step is to copy that formula from cell `C9` to cell `C10`. With any luck Excel will guess what we want to do, change `B9` to `B10`, compute

```
=C4*B10+C5
```

and display `44.11`. But that's not what happens! Excel shows `4469.79` instead! If you look at the contents of cell `C10` you will find

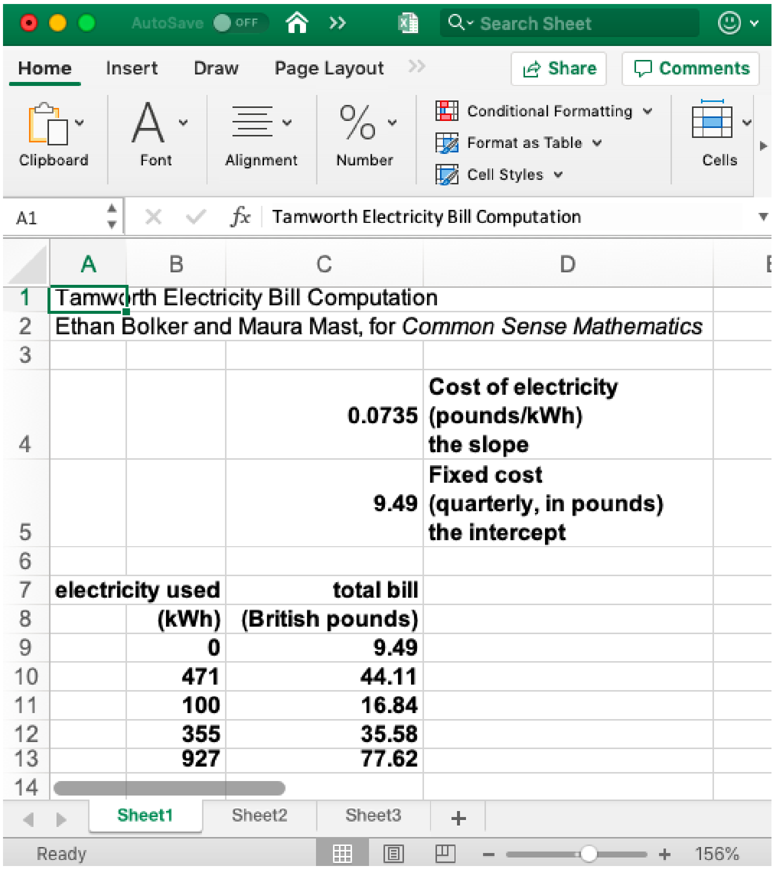


Figure 7.3. Tamworth electricity bill

```
=C5*B10+C6
```

so Excel added 1 to the row numbers for cells `C4` and `C5` as well as for `B9`. There's nothing in cell `C6`. Excel treats that as a zero and adds it to 471×9.49 to get 4469.79.

That's not what we want. Changing `B9` to `B10` is right, but we want Excel to leave the references to cells `C5` and `C6` alone. The trick that makes that happen is to put a `$` in front of the `5` and the `6`. This is not something you could have figured out. There's no particular reason why this trick should work. Just remember it. The right formula to use in cell `C9` is

```
=C$4*B9+C$5 .
```

When we copy that formula from `C9` to cells `C10:C13` we get Figure 7.3.

Figure 7.4 is a screen shot of the same spreadsheet — after we asked Excel to show the formulas for each cell instead of the values.

In Chapter **??** we learned how to use Excel to draw bar charts and histograms so that we could visualize data organized into categories. The x-axis displayed category names, with

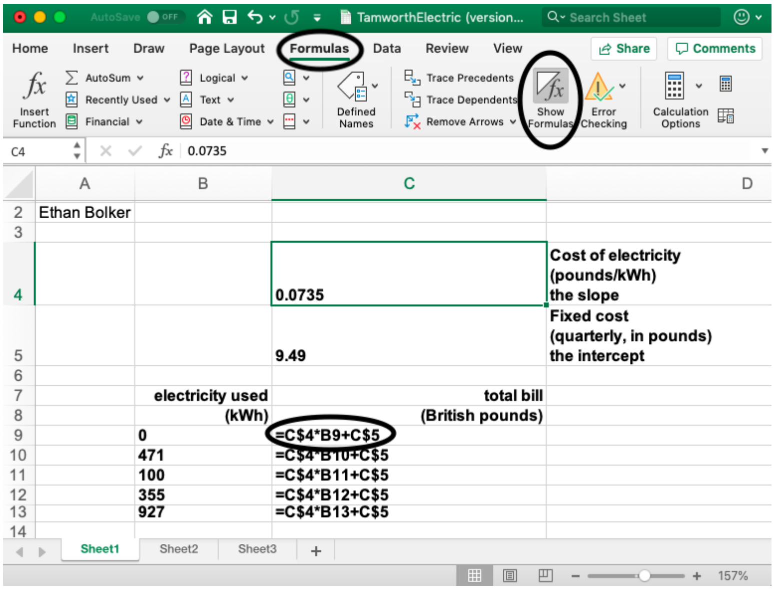


Figure 7.4. Tamworth electricity bill — formulas

corresponding values on the y-axis. That won't work for the data in Figure 7.3, since there both the x- and y-axes have numerical values. Instead, after selecting cells `B7:C13` we must ask Excel for a chart of type `XY(Scatter)`. Figure 7.5 shows the result.

The graph is a straight line — that's why the function is called "linear". The slope tells us how steep the line is and the intercept tells us where it crosses the vertical axis — in this case at the value £9.49, the total bill when you use no electricity at all.

Excel will let you change the type of a chart once it's built. If you change the chart in Figure 7.5 to a `Line Chart` Excel will use the data in the column `B` as category labels rather than as the numbers of kilowatt-hours. It will space them evenly along the x-axis, whatever their values, and draw the nonsense you see in Figure 7.6.

If you change the chart type to scatter you get two scatters, one for each column. You can get Figure 7.5 only if you start with a scatter plot. If you select the two columns of data and build a line chart first, things are even worse. Excel thinks each row is a category for which you have two pieces of data. It labels the categories 1, 2, . . . and shows a line for each. Try this and see what happens.

7.5 Which truck to rent?

Table 7.7 shows the cost of renting a truck for one day in Boston in March of 2015. Four companies offer equivalent models. Which one should you choose?

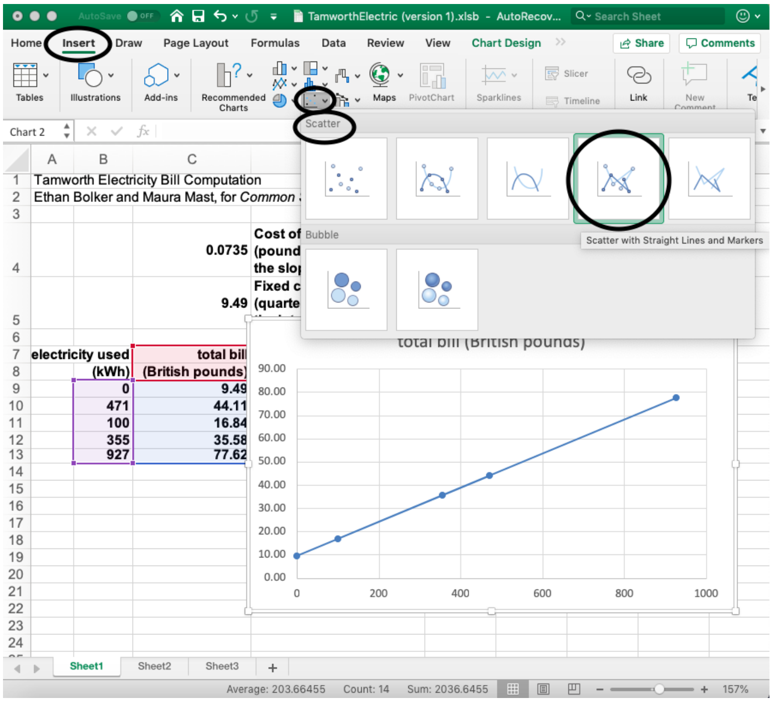


Figure 7.5. Tamworth electricity bill — chart

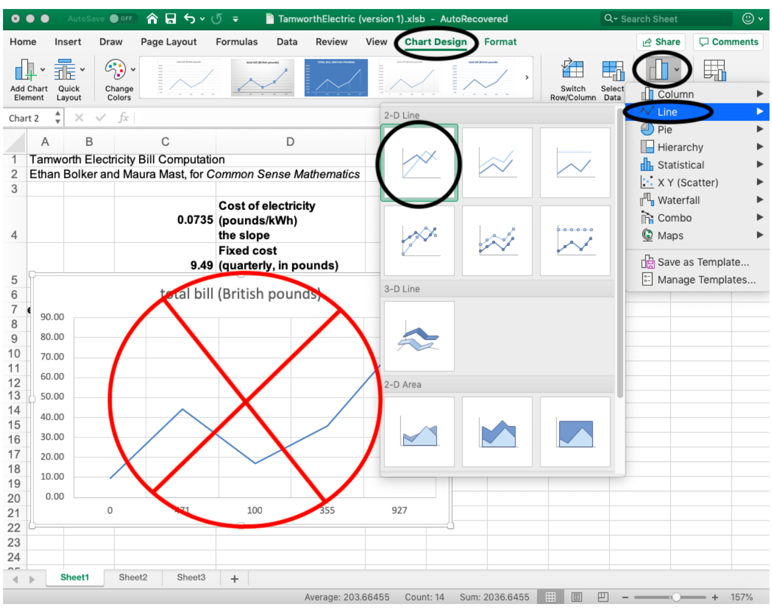


Figure 7.6. How NOT to draw a line

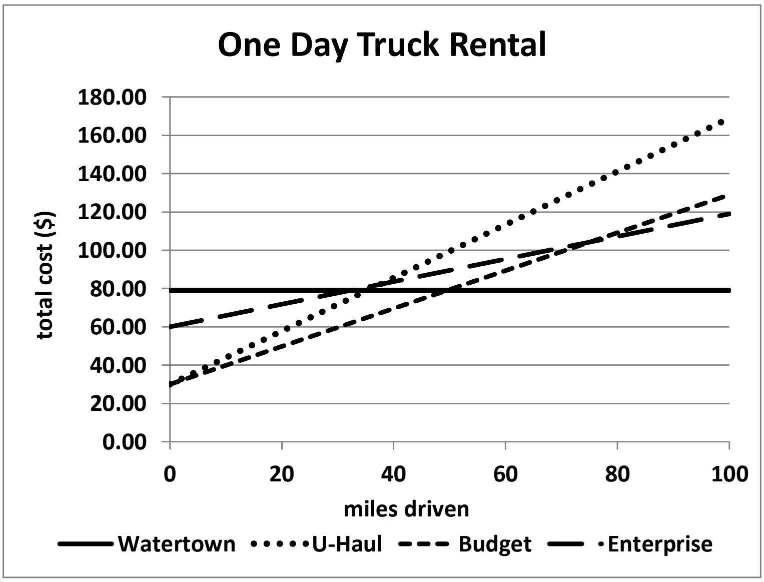


Figure 7.8. Comparing truck rental costs

	Watertown	U-Haul	Budget	Enterprise
fixed cost	79	29.95	29.95	59.99
\$/mile	0	1.39	0.99	0.59

Table 7.7. One day truck rental costs

It's clear from the numbers that for a very few miles Budget will be cheapest, since it's tied with U-Haul for the lowest fixed cost and charges less per mile. For a really long move Watertown will be best since there is no mileage charge. The Excel chart in Figure 7.8 tells the whole story. Budget is cheapest up to about 50 miles. For longer trips, choose Watertown. It never makes financial sense to rent from U-Haul or Enterprise.

The figure is a good reminder of the meaning of slope and intercept for straight line graphs. A line crosses the vertical axis at the intercept. In this example intercepts represent the fixed costs. The units of the intercept are dollars — the units on the y-axis. The slope of a line measures how steep it is. That's particularly visible when you compare U-Haul and Budget, which have the same intercept but different slopes — 1.39 dollars/mile and 0.99 dollars/mile. The units of the slope are always (units on y-axis)/(units on x-axis), The line for Watertown is horizontal since its slope is 0 dollars/mile.

The spreadsheet with that figure is at `TruckRental.xlsx.` Figure 7.9 shows how we arranged the formulas in the spreadsheet to compute the total cost for each company. Column `A` lists the mileages we're considering, from 0 in cell `A11` to 100. The formula `=A11+10` in cell `A12` fills column `A` when we copy it to cells `A13:A21`. The formula in cell `B11` is

```
=B$7+$A11*B$8 .
```

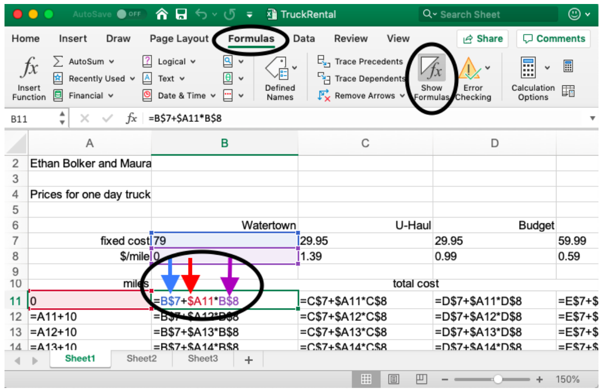


Figure 7.9. Computing truck rental costs

It uses three $ signs to keep Excel from adjusting references for rows `7` and `8` and for column `A`. That allowed us to copy it to all of the range `B11:E21`.

The problem we've just solved is typical of situations where you have to decide among options, some with small startup cost but a high ongoing rate, others the reverse. Here are some examples:

- Insulate your house (high initial investment compared to doing nothing) in order to pay less for heat in the winter (lower rate for use).
- Buy a hybrid instead of a conventional car (higher initial cost, lower rate of fuel consumption).
- Buy energy efficient light bulbs (more expensive to start with, but they use less electricity to run),
- Select a phone plan with unlimited text messaging (more expensive than pay-as-you-text, but the slope ($ per text) is zero).

Each of these can be thought of as looking at linear equations to see where their graphs cross. You can do that by building a table of values or by drawing the graphs (in Excel, or with pencil and paper) or by writing down the equations and solving them with remembered algebra, or by guess-and-check.

But you can't rely on just this mathematics to make a decision. There are always other important things to think about. How long does the more expensive purchase last? Can you afford the initial high payment? If so, what else might you rather do with that money? Do you need to take depreciation or inflation into account?

7.6 Energy and power

How much electrical energy does a 100 watt light bulb use? That depends on how long it's on. When it's switched off, it doesn't use any at all. If it's on for two hours it must use

Energy transferred	=	power	×	time
(kilowatt-hour, kWh)		(kilowatt, kW)		(hour, h)

Figure 7.10. How much electrical energy?

twice as much as it does in one hour. Figure 7.10 (from the second page of the Tamworth bill) shows the proportion lurking there.

The second line in that figure displays the units for the quantities in the first line. Time is measured in hours, of course. Power is measured in kilowatts. Energy is measured in kilowatt-hours: the product of the units for power and for time.

That tells us right away that energy and power are not the same thing. Comparing the figure to Equation 7.1, you can see that energy is like distance — a thing that's consumed or traveled, while power is like speed — the rate at which energy is used or distance covered.

If you turn a 100 watt light bulb on for 2 hours the formula in Figure 7.10 tells you how much electrical energy you use:

$$100 \text{ watts} \times 2 \text{ hours} = 200 \text{ watt-hours} \quad (7.2)$$

$$= 0.2 \text{ kilowatt-hours} \quad (7.3)$$

(7.2) is just multiplication. (7.3) changes watt-hours to kilowatt-hours.

How much does it cost to leave a 40 watt bulb on all the time in your basement for a year? There are about 9,000 hours in a year. That is 9 kilo-hours, so you'll use about

$$40 \text{ watts} \times 9 \text{ kilo-hours} = 360 \text{ kilowatt-hours.}$$

If you pay $0.10 per kwh for electricity it will cost you about $36 per year to guarantee that you don't fall down the basement steps in the dark.

The electrical energy that flows through the wires in your house to your appliances probably comes to you from a power plant, which might be burning coal or natural gas or extracting energy from nuclear fuel. (You might have a wind turbine in your neighborhood, or a hot spring, or solar panels on your roof, but these are unlikely power sources for most people.) So power plants produce energy, not power. The power of a power plant is the rate at which it can produce energy, so "energy plant" would be a better name than "power plant".

The website for Chicago's Cook Nuclear Plant says that

> The 1,048 net megawatt (MW) Unit 1 and 1,107 net MW Unit 2 combined produce enough electricity for more than one and one half million average homes. [R2]

Let's check this. The combined total power is 2,155 megawatts. This is the rate at which that plant produces electrical energy when it is running at full power. (When it's not running it's still just as powerful, but not producing any energy.) When it's running, how many average homes could it produce electricity for?

If the Cook plant ran all year (about 9,000 hours) it would produce

$$\begin{aligned} 2,155 \text{ megawatts} \times 9,000 \text{ hours} &\approx 18,000,000 \text{ megawatt-hours} \\ &= 18,000,000,000 \text{ kilowatt-hours} \end{aligned}$$

of electrical energy. Googling "average household electricity usage" finds

6,000 kwh per household per year for 3 residents average per household

from `www.physics.uci.edu/~silverma/actions/HouseholdEnergy.html` . The source is a physics professor's website, so it's probably reliable. At 6,000 kwh per household per year Cook could power 3 million homes. The quotation claims half that, so it's clearly in the right ballpark. The 6,000 kwh per household per year is a southern California average — households in northern Illinois might well use more electricity.

Energy comes in many forms besides electric. The Cook plant converts the energy in its nuclear fuel to electricity. Driving a car uses the energy stored in the gasoline. Running a marathon uses the energy in the food you eat. Each form of energy has its own units. We've seen that electrical energy is measured in kilowatt-hours. If you cook on a gas stove, the energy in the gas is measured in *therms*. The energy in the oil that heats your house is measured in *British Thermal Units* or Btus. The energy in the food you eat is measured in *calories*. Physicists measure energy in *ergs* or *joules*; you rarely see those units in everyday life. You can look up conversion factors for these units — for example, the energy in a barrel (42 gallons) of oil is about 5.8 million Btu, which is equivalent to 1700 kilowatt-hours. So it would take about a fifth of a barrel to keep that 40 watt light bulb burning for a year.

Converting among the units for energy is just like converting among the units for length (meters, feet, yard, miles, ...). You can use a table, an online calculator like the one at the National Institute of Standards and Technology (`physics.nist.gov/cuu/Constants/energy.html`) or the Google calculator.

Possibly the most interesting energy conversion is the one that Einstein discovered in 1905: mass and energy are the same thing, measured in different units. The conversion factor is the square of the speed of light — hence the famous equation

$$e = mc^2.$$

To see that at work, look again at the yearly energy output of the Cook plant. The Google calculator tells us that

$$\boxed{18\,000\,000\,000 \text{ kilowatt-hours} = 6.48 \times 10^{16} \text{ joules}} \; .$$

The National Institute of Standards and Technology website says that corresponds to a mass of about 0.72 kg, which is 720 grams. That means just about 1.6 pounds of matter must be converted to energy to power millions of Chicago homes for a year. The Google calculator does not know Einstein's equation, so it wouldn't convert kwh to grams directly!

Bracket ($)	Marginal Tax Rate (%)
0 – 9,700	10
9,701 – 39,475	12
39,476 – 84,200	22
84,201 – 160,725	24
160,726 – 204,100	32
204,101 – 510,300	35
510,301 –	37

Table 7.11. 2019 single taxpayer brackets and rates

7.7 Federal payroll taxes

Income tax.

Taxes are a part of life (the only other certainty is death), so it's only common sense to learn how they work. In Section 7.1 we studied sales taxes. Cities and states collect them; they are computed as a percentage of the purchase price. In this section we'll explain two important federal taxes that depend on your income, not on how you spend it.

Federal income tax is not simply a proportion of your income. It's a *progressive graduated tax*. When you make more money you not only pay more tax, some of your income may be taxed at a higher rate. Table 7.11 shows the 2019 *tax brackets* for single taxpayers.

That tells you that the first $9,700 of your income is taxed at 10%. If you make exactly that much, you pay $970 in tax. If you make more, the extra income is taxed at a higher rate — you have moved to a higher *tax bracket*. For example, if you make between $9,700 and $39,475 you will pay $970 for the first $9,700 and 12% of the amount you earn over $9,700. If you earn more than $39,475 you start paying at a 22% rate on the extra.

Let's try an example. If your taxable income is $50,000, then your total tax is

$$\begin{aligned} \text{total tax} &= 0.10 \times \$9,700 + 0.12 \times (\$39,475 - \$9,700) \\ &\quad + 0.22 \times (\$50,000 - \$39,475) \\ &= \$970.00 + \$3573.00 + \$2315.50 \\ &= \$6858.50\ . \end{aligned} \tag{7.4}$$

Note carefully that when you are in a higher tax bracket the higher rate applies only to the extra income. The taxpayer in this example is in the 22% bracket, but that rate applies only to her earnings in that bracket.

Individual Taxpayers

If Taxable Income Is Between:	The Tax Due Is:
0 - $9,700	10% of taxable income
$9,701 - $39,475	$970 + 12% of the amount over $9,700
$39,476 - $84,200	$4,543 + 22% of the amount over $39,475
$84,201 - $160,725	$14,382.50 + 24% of the amount over $84,200
$160,726 - $204,100	$32,748.50 + 32% of the amount over $160,725
$204,101 - $510,300	$46,628.50 + 35% of the amount over $204,100
$510,301 +	$153,798.50 + 37% of the amount over $510,300

Figure 7.12. 2019 tax brackets [R3]

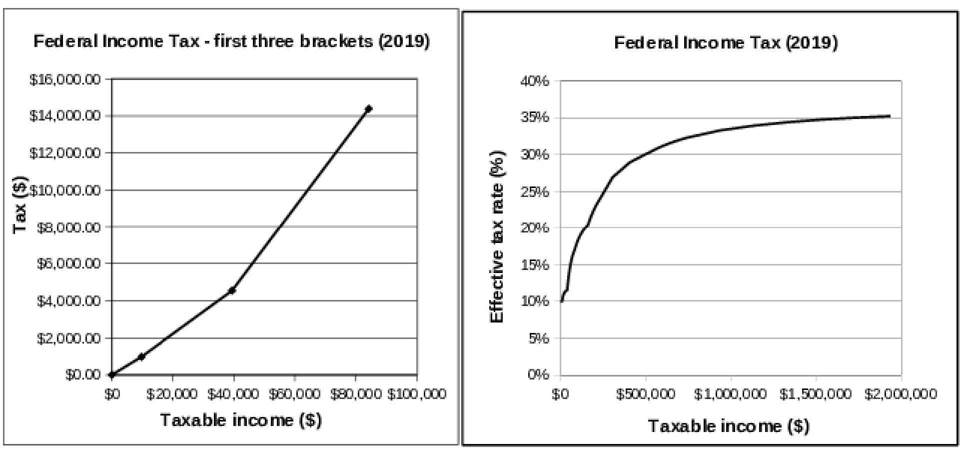


Figure 7.13. 2019 single taxpayer tax liability

Figure 7.12 explains this rule in another way.

The first graph in Figure 7.13 from the spreadsheet `GraduatedTax2019.xlsx` shows that the dependence of tax on income is *piecewise linear* — built from pieces of straight lines that become steeper as income increases.

The second graph shows the effective tax rate — the percentage of your income you pay in federal income tax. In Equation 7.4 we found that the total tax on a taxable income of $50,000 was $6858.50. The effective tax rate is $\$6858.50/\$50{,}000 = 13.72\%$. This is a weighted average of the three bracket rates 10%, 12% and 22%, with weights the amount of income taxed at each rate. The effective tax rate is less than the rate in your top bracket because you pay at a lower rate on the first part of your income. The effective rate does not reach 35% until about $2 million in income — well into the top 37% bracket.

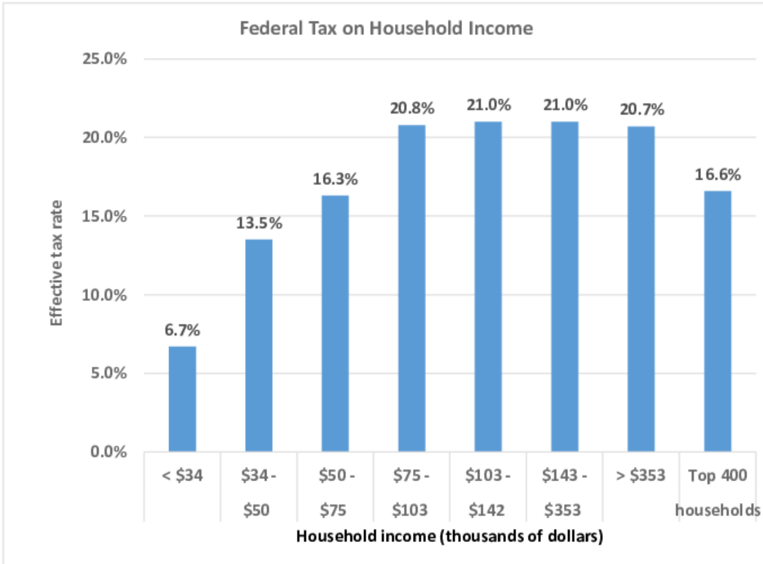


Figure 7.14. Actual effective federal tax rate by income, 2007 [R4]

In fact, the actual effective tax rate is lower than this for wealthier households because income tax is collected only on income from wages and earnings. Income from capital gains — returns on investment — is taxed at a lower rate. Figure 7.14 shows the actual effective federal tax rate by total household income for the year 2007. The rate for the wealthiest households was just 16.6% — less than half the 35% rate for the top income tax bracket that year. It's reasonable to assume that a similar discrepancy is still true.

Social security.

Social security tax payroll deductions show up labelled "FICA" on your pay stub. That acronym is from the "Federal Insurance Contributions Act". Those taxes pay for Social Security and Medicare.

In 2019 the starting tax rate was 6.2% for Social Security and 1.45% for Medicare. The Social Security tax is collected only on the first $132,900 of your earnings. Up to that income level the combined rate is 7.65%.

When your earnings exceed $132,900$ you pay no more Social Security tax, but you continue to pay Medicare tax at the 1.45% rate. When your income reaches $200,000 the Medicare rate increase to 2.35% on the amount over $200,000.

The actual rules are a little more complicated. First, the tax applies only to wages. Other income (like stock dividends or interest) are not subject to this tax. Second, the real rates are twice the quoted amounts, but your employer is required to pay half. If you're self-employed you pay it all.

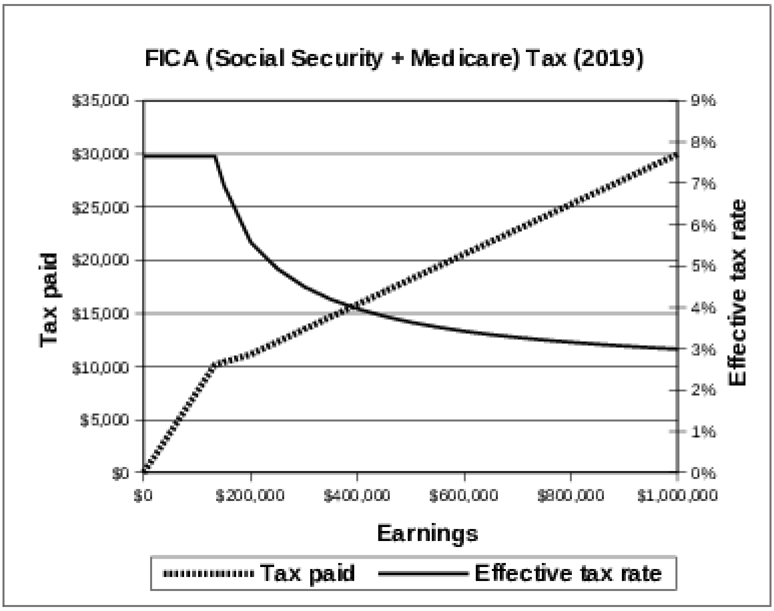


Figure 7.15. FICA (Social Security and Medicare) tax

If you earn $500,000 your FICA tax is

$$0.062 \times \$132,9000 + 0.0145 \times \$200,000 + 0.0235 \times (\$500,000 - \$200,000)$$
$$= \$18,190.$$

Since you pay no Social Security on wages over $132,900 the percentage of your earnings collected for FICA taxes decreases as your earnings increase even though you still pay for Medicare. So FICA taxes are *regressive*. Up to $139,700 the effective rate is 7.45%. For $500,000 the effective rate is just $\$18,190/\$500,000 = 3.64\%$. For higher incomes, the effective rate is even smaller. At huge incomes it levels off at the top Medicare rate of 2.35%. Figure 7.15 from spreadsheet `SocialSecurityTax2019.xlsx` shows the amount of FICA tax paid and the decreasing effective tax rate as a function of FICA earnings.

Income tax history.

For complex historical, legal and political reasons the Supreme Court rejected the first attempts to collect an income tax. This Constitutional amendendment, ratified in 1909, made that kind of tax legal.

> Amendment XVI
> The Congress shall have power to lay and collect taxes on incomes, from whatever source derived, without apportionment

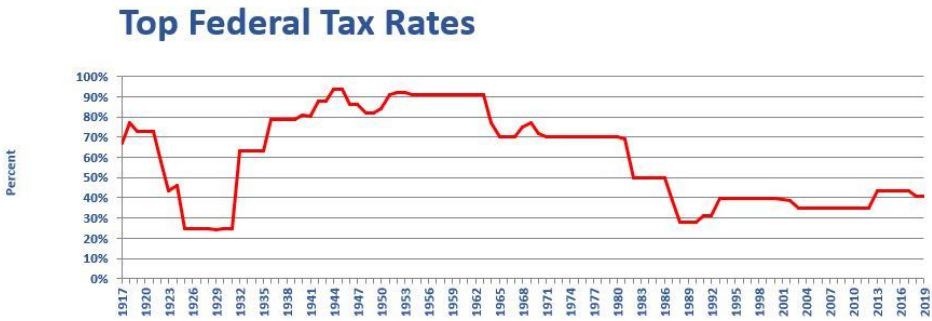


Figure 7.16. Historical top tax bracket rate [R6]

> among the several States, and without regard to any census or enumeration. [R5]

The first federal income taxes were collected in 1913. Figure 7.16 shows the significant fluctuations in the rate for the top bracket through 2019. They have been near their historic lows since the late 1980's.

To assess the social and economic impact of the changes in income tax rates would require much more time and knowledge than we can offer here. The spreadsheet `Federalindividualratehistory.xlsx` contains a complete history of income tax brackets and rates from the inception of the income tax in 1913 through its hundredth anniversary in 2013, in both dollars current in each year and adjusted for inflation (2012 dollars).

Even in years when Congress does not revise the tax code, the IRS routinely adjusts the brackets (not the rates) to take inflation into account. If that were not done then salaries increased by inflation would move people into higher brackets even when their increased wages did not correspond to increased purchasing power.

Figure 7.17 shows the brackets for the tax year 2018. You can check that the 2019 brackets are all just about 2% larger than these.

7.8 Exercises

Exercise 7.8.1. [U][Section 7.2][Goal 7.2] [Goal 7.3] Your electricity bill.

Verify the computations on your current electricity bill, either with a calculator or by modifying the Tamworth bill spreadsheet at `TamworthElectric.xlsx`.

Individual Taxpayers

If Taxable Income Is Between:	The Tax Due Is:
0 - $9,525	10% of taxable income
$9,526 - $38,700	$952.50 + 12% of the amount over $9,525
$38,701 - $82,500	$4,453.50 + 22% of the amount over $38,700
$82,501 - $157,500	$14,089.50 + 24% of the amount over $82,500
$157,501 - $200,000	$32,089.50 + 32% of the amount over $157,500
$200,001 - $500,000	$45,689.50 + 35% of the amount over $200,000
$500,001 +	$150,689.50 + 37% of the amount over $500,000

Figure 7.17. 2018 tax brackets [R7]

If you don't have a current electricity bill, check the website for your local electric company, which probably provides a sample bill you can use instead.

Exercise 7.8.2. [U][Section 7.2][Goal **??**] [Goal 7.2] Electricity costs now and then and here and there.

Compare residential electricity cost in Boston in 2007 (the date of the NStar bill in Figure 7.2) to the cost where you live today. Your answer should take inflation into account.

If you have a current electricity bill, use it. If you don't, try to find the fixed monthly cost and the cost of electricity in $/kwh from your local electric company. Perhaps their website has that information.

Exercise 7.8.3. [R][S][Section 7.2][Goal 7.2] [Goal 7.3] How much electricity does it use . . . ?

The document containing the Tamworth bill asks how much electricity various appliances use.

(1) Calculate how much electricity is consumed by a
   (a) 100w lamp on for 2 hours.
   (b) 500w TV on for 5 hours.
   (c) 2kw kettle on for half an hour.
   (d) 10w electric blanket on for 15 minutes.
(2) Compare the electricity used by
   (a) 2kw heater for 2 hours or 3 kw heater on for 3 hours.

(b) 900w toaster for 15 minutes or 2kw grill for 10 minutes.
(c) 100w radio for 2 hours or 500w radio for 45 minutes.
(d) Which appliance would be cheaper to use in each case?

(1) Calculate how much electricity is consumed by a

(a) 100w lamp on for 2 hours.

$$100\text{w} \times 2 \text{ hours} \times \frac{\text{kilo}}{1000} = 0.2 \text{ kilowatt-hours.}$$

(b) 500w TV on for 5 hours.

$$500\text{w} \times 5 \text{ hours} \times \frac{\text{kilo}}{1000} = 2.5 \text{ kilowatt-hours.}$$

(c) 2kw kettle on for half an hour.
1 kilowatt-hour.

(d) 10w electric blanket on for 15 minutes.

$$10\text{w} \times 0.25 \text{ hours} \times \frac{\text{kilo}}{1000} = 0.0025 \text{ kilowatt-hours.}$$

(2) Compare the electricity used by

(a) 2kw heater for 2 hours or 3 kw heater on for 3 hours.
Clearly 4kw hours is less than 9 kw hours.

(b) 900w toaster for 15 minutes or 2kw grill for 10 minutes.
It's easier to do this one if I leave the times in minutes.
$0.9 \times 15 = 1.35$ kw-minutes is less than $2 \times 10 = 20$ kw-minutes.

(c) 100w radio for 2 hours or 500w radio for 45 minutes.
I'll do this one in minutes too, and use watts rather than kilowatts.
$100 \times 120 = 12,000$ watt-minutes is less than $500 \times 45 = 22,500$ watt-minutes.

(d) Which appliance would be cheaper to use in each case?
In each case the first one is cheaper - the amount of electricity is about half.

Exercise 7.8.4. [S][Section 7.2] [Goal 7.1][Goal 7.7] Direct current.

In an article from *The New York Times* on November 17, 2011 headlined "From Edison's Trunk, Direct Current Gets Another Look" you can read that

> In a data center redesigned to use more direct current, monthly utility bills can be cut by 10 to 20 percent, according to Trent Waterhouse, vice president of marketing for power electronics at General Electric. Verizon Communications, a G.E. customer, expects to save 1 billion kilowatt-hours a year from a nationwide retrofit of its data centers, which translates to roughly enough to power 77,000 homes. [R8]

(a) How many watt-hours is a billion kilowatt-hours?

(b) About how much electricity is Verizon using now, given that they hope to save a billion kilowatt-hours per year?

(c) Verify the estimate that those billion kwh are "roughly enough to power 77,000 homes". (You won't find anything useful in the original article. How else will you search?)

(a) How many watt-hours is a billion kilowatt-hours?

Since "kilo" means "multi ply by 1000", a billion kilowatt-hours is (a billion times a thousand) watt-hours. That's a trillion watt-hours, or 10^{12} watt-hours, or a terawatt-hour. If you have to write the zeroes, it's 1,000,000,000,000 watt-hours.

(b) About how much electricity is Verizon using now, given that they hope to save a billion kilowatt-hours per year?

If the billion kwh saved were 10% of what they are using now, that would mean they were using 10 billion kwh now. If the billion kwh were 20% of current use, that would make current use 5 billion kwh. So they are using between 5 and 10 billion kwh now.

(c) Verify the estimate that those billion kwh are "roughly enough to power 77,000 homes". (You won't find anything useful in the original article. How else will you search?)

The website `www.eia.gov/tools/faqs/faq.cfm?id=97&t=3` says that

> In 2010, the average annual electricity consumption for a U.S. residential utility customer was 11,496 kwh, an average of 958 kilowatthours (kwh) per month. Tennessee had the highest annual consumption at 16,716 kwh and Maine the lowest at 6,252 kwh.

Using the figures in the original quotation

$$\frac{1 \text{ billion kwh}}{77{,}000 \text{ homes}} = 12{,}987.013 \frac{\text{kwh}}{\text{home}},$$

which is close enough to the government's figure of 11,496 kwh/home.

Exercise 7.8.5. [S][Section 7.3][Goal 7.2] How hot was it?

Figure 7.18 shows a weather forecast for Hamilton, Ontario, Canada. The temperature there is displayed in degrees Celsius, marked °C.

(a) Look up the linear relationship for converting temperatures measured on the Celsius scale to temperatures on the Fahrenheit scale.

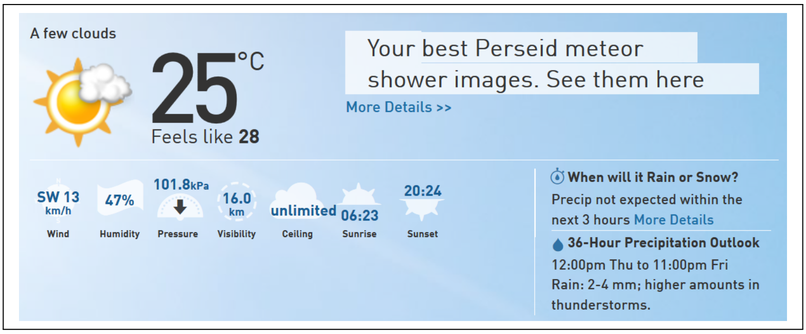


Figure 7.18. Weather in Canada

(b) Use the formula you found to calculate the temperature in degrees Fahrenheit in Hamilton, Ontario on Thursday August 13, 2015 at 12:45.

(c) Check your answer using the Google calculator.

(d) What does "Wind: SW 13 km/h" in the figure mean?

(e) What does "Pressure: 101.8 kPa" in the figure mean?

(f) What is the Perseid meteor shower?

(a) Look up the linear relationship for converting temperatures measured on the Celsius scale to temperatures on the Fahrenheit scale.

The formula is

$$F = \frac{9}{5}C + 32.$$

(b) Use the formula you found to calculate the temperature in degrees Fahrenheit in Hamilton, Ontario on Thursday August 13, 2015 at 12:45.

$$\frac{9}{5}25 + 32 = 77.$$

(c) Check your answer using the Google calculator.

I did. Same answer.

(d) What does "Wind: SW 13 km/h" in the figure mean?

Wind blowing from the southwest at 13 kilometers per hour.

(e) What does "Pressure: 101.8 kPa" in the figure mean?

It's a measure of air pressure. "Pa" is a "Pascal", a unit of pressure. The k is kilo, of course. So the pressure was 101.8 thousand Pascals.

(f) What is the Perseid meteor shower?

Every year in August the Earth passes through a cloud of rocky junk in space that leads ot a lot of meteors as the junk burns up in the atmosphere.

Exercise 7.8.6. [S][Section 7.3][Goal 7.2] [Goal 7.3][Goal 7.4] The Jollity Building.

In Exercise **??** there's an implicit linear model for Mr. Ormont's weekly income. Write the linear equation for that model. Clearly identify the independent and dependent variables and the units for the slope and the intercept.

The clue to finding the linear equation is the phrase "fifty dollars a week and 2 percent of the total rents." I'll let W stand for Mr. Ormont's weekly income and R for the weekly rent. Then

$$W = \$50 + 0.02R.$$

The constant amount (the intercept) is \$50. He gets that much even if the building is empty. The slope is 0.02. Its units are

$$\frac{\text{dollars for Mr. Ormont}}{\text{dollar of weekly rent}}.$$

Exercise 7.8.7. [S][Section 7.3] [Goal 7.2][Goal 7.3] Newton trees .

In an article in the April 2012 issue of the Newton Conservators newsletter you can read that

> In the early 1970s there were approximately 40,000 trees lining the streets of Newton. Today, that number is about 26,000 — a 35% loss. The current annual rate of decline is about 650 trees per year. At this rate, if unchecked, public street trees would diminish to approximately 10,000 within a generation (25 years), and in 40 years, public street trees would no longer be part of the Newton landscape. [R9]

(a) Check the arithmetic that leads to the claimed "35% loss".

(b) Check the arithmetic that leads to a "current annual rate of 650 trees per year".

(c) Check the predictions in the last sentence. Are they likely to come to pass?

(d) Write the equation for the linear model implicit in this quotation (use years since 2012 as the independent variable). Identify the slope and the intercept, with their units.

(a) Check the arithmetic that leads to the claimed "35% loss".

The 1+ trick tells me 40K $times 0.65 = 26$K.

(b) Check the arithmetic that leads to a " current annual rate of 650 trees per year".

If I take "the early 1970s" to be 1972 then the interval is 40 years.

$$\frac{14,000 \text{ trees}}{40 \text{ years}} = 350\frac{\text{trees}}{\text{year}}.$$

That doesn't match the figure in the article. Perhaps the rate of loss is larger in more recent years.

(c) Check the predictions in the last sentence. Are they likely to come to pass?

Using the rate of 650 trees per year, the loss in 25 years would be $25 \times 650 \approx 16,000$ trees. Subtracting that from the current 26,000 trees would indeed leave just 10,000.

In another 15 years the loss would be just about all the 10,000 remaining trees.

(d) Write the equation for the linear model implicit in this quotation (use years since 2012 as the independent variable). Identify the slope and the intercept, with their units.

The independent variable is Y, years since 2012. The dependent variable is T, the number of trees. The equation is

$$T = 26,000 - 650Y.$$

The intercept is 26,000 trees, the slope is -650 trees/year.

Exercise 7.8.8. [R][S][A][Section 7.4] [Goal 7.2][Goal 7.3][Goal 7.5] Apple time.

Kuipers Family Farm (`www.kuipersfamilyfarm.com/apple-orchard/`), in Maple Park, IL, charged \$10 per person for admission to the apple orchard during the 2018 apple season. This included a $\frac{1}{2}$ peck bag of apples and a hayride to the orchard. Visitors could pick additional peck bags of apples for \$15 each. (A bag of apples in the grocery store usually contains $\frac{1}{2}$ peck.)

(a) If you pick that additional peck of apples, how much do you pay?

(b) If you pick two additional pecks of apples, how much do you pay?

(c) If you just enjoy the free cider, your $\frac{1}{2}$ peck of apples and the sunshine, what do you pay?

(d) Write a linear function that shows how the total cost of the farm visit depends on the how many pecks of apples you pick. Identify the slope and intercept, with their units.

(e) Build an Excel spreadsheet to compute this function and use it to check the values you worked out by hand above. Include a chart in your spreadsheet.

(f) Use your spreadsheet to calculate your apple cost (in \$/peck) when you pick no extra pecks, then when you pick 1, 2 or 10 extra pecks.

I found the function first. If C is the total cost of the visit when you pick A pecks of apples then C depends on A this way:

$$C = 15A + 10.00.$$

That's a linear function with slope 15 \$/peck and intercept \$10.00.

I used the function to find the following answers.

(a) If you pick that additional peck of apples, how much do you pay?
$25.00.

(b) If you pick two additional pecks of apples, how much do you pay?
$40.00.

(c) If you just enjoy the free cider, your $\frac{1}{2}$ peck of apples and the sunshine, what do you pay?
$19.00. Didn't need the formula for this one.

(d) Write a linear function that shows how the total cost of the farm visit depends on the amount of apples picked. Identify the slope and intercept, with their units.
Did that first.

(e) Build an Excel spreadsheet to compute this function and use it to check the values you worked out by hand above.
See `../Answers/U-PickApplesSolution.xlsx`

(f) Use your spreadsheet to calculate your apple cost (in $/peck) when you pick no extra pecks, then when you pick 1, 2 or 10 extra pecks.
The number of pecks of apples you go home with is $A+\frac{1}{2}$. I used the spreadsheet to compute $C/(A+\frac{1}{2})$ for $A=0,1,2$ and 10 and found the cost per peck to be $\$20.00, \$16.67, \$16.00$ and $15.24. I'm not surprised that the more you pick the closer the cost per peck is to $15.

Exercise 7.8.9. [S] [A][Section 7.5] [Goal 7.2][Goal 7.5] Comparing telephone calling plans.

A cell phone company has introduced a pay-as-you-go price structure, with three possibilities.

Plan 1	$10 a month	10 cents per minute
Plan 2	$15 a month	7.5 cents per minute
Plan 3	$30 a month	5 cents per minute

(a) For each plan, find a linear function that describes how the total cost for one month depends on the number of minutes used.

(b) Construct a table in Excel showing the total cost for one month for each of the three plans. Organize your data this way:

- Create a sequence of cells in column A for the various possible numbers of minutes. Label that column. Start with 0 minutes. What's a good step to use? What's a reasonable place to stop?

- Use columns `B`, `C` and `D` for each of the three plans. The fixed charge and charge per call should be in cells in those columns too, so you can use the same formula everywhere in the data table. (That will call for clever use of the `$` to keep Excel from changing row numbers and column letters when you don't want it to.)

(c) Use Excel to draw one chart showing how the monthly bill (y-axis) depends on the number of minutes you use the phone (x-axis) for all three plans.

(d) Write a paragraph explaining to your friend how she should go about choosing the plan that's best for her.

[See the back of the book for a hint.] Finding the places where the lines in your Excel chart cross is the key to the last part of the problem.

(a) For each plan, find a linear function that describes how the total cost for one month depends on the number of minutes used.

Let B represent the monthly cost, in dollars, and M the number of minutes used. Then the three linear functions are

$$B = 0.10M + 10,$$
$$B = 0.075M + 15,$$
$$B = 0.05M + 30.$$

(b) Use Excel to draw one chart showing how the monthly bill (y-axis) depends on the number of minutes you use the phone (x-axis) for all three plans.

See the spreadsheet at `../Answers/PhoneCostsSolution.xlsx`.

(c) If you talk for less than 200 minutes, choose Plan 1. If you talk for more than 600, choose Plan 3. For anything in between, choose Plan 2. You can check my work by looking at the spreadsheet — the conclusion is plain from both the table and the chart.

Exercise 7.8.10. [S][Section 7.5][Goal 7.2] [Goal 7.5] Prepaid phones.

Until summer 2012, if you wanted an iPhone you needed to lock into a two-year contract. Then some mobile companies started selling the iPhone and letting you choose your own plan, with no contract. You can use the ideas from this chapter to make a quantitative comparison (we'll let you decide what other factors, such as paying a lot up front, matter in your decision).

Virgin Mobile began selling a 16GB iPhone 4S for \$649.99, with no plan or contract. They offered a \$55 per month "unlimited" data plan (in fact it was not unlimited, as once you go past 2.5GB of data they slowed the phone speed down considerably). If you purchased the phone through Sprint, it cost just \$149.99. However, you had to sign up for a two-year contract. The least expensive option was \$79.99 with what they called unlimited data.

(a) For each plan, find a linear function that shows how the total cost of the phone depends on the number of months you have it.

(b) Build a spreadsheet in Excel using your functions and fill in the cost of the two different options over several months.

(c) Graph the data in your spreadsheet.

(d) Write a short statement comparing the two plans. Clearly the Virgin Mobile plan is more expensive at first. When does it become the less expensive plan? If you had to choose one of the plans, which one would you choose and why? What other factors would you consider?

The answers to all the questions are in the spreadsheet at `../Answers/PrepaidPhonesSolution.`

Exercise 7.8.11. [S][Section 7.5][Goal 7.3] [Goal 7.5] Hybrid payback.

The "Best & Worst Cars 2011" issue of *Consumer Reports* provides the following data for new Toyota Camrys:

	conventional	hybrid
cost	\$19,720	\$26,575
fuel economy	26 MPG	34 MPG

Assume gasoline costs \$3.50/gallon.

(a) Questions about the conventional Camry.

- (i) Once you own the car, how much does it cost to run, in dollars per mile? Does your answer make sense?
- (ii) Calculate the total cost (purchase plus gasoline) to drive the conventional Camry 10,000 miles.
- (iii) Write the linear equation that computes the total cost C of driving the conventional Camry M miles.
- (iv) Identify the slope and the intercept of this equation, with their units.

(b) Open the spreadsheet `ConventionalvsHybrid.xlsx`. Enter the numerical data from the table and the cost of gasoline in the appropriate cells. What formula should you enter in cell `C15` to check your answer to part (i)?

(c) Copy your formula to cells `B14:D29` to fill in the table. Where must you add \$ signs to keep Excel from changing row and column references?

Create a properly formatted and labelled chart in Excel showing how the cost of driving each car depends on the number of miles driven. Use your graph along with the table to answer the following questions.

(d) If you drive 120,000 miles will you recover in gas savings the extra initial cost of the hybrid? Write a complete sentence or two and use appropriate precision for the numbers you use to make your argument.

(e) When will you recover the extra initial cost in gas savings if the government (re)instates a $3,000 tax rebate for hybrid purchases?

(f) With the original initial costs, how much would the price of gasoline have to be in order for the breakeven point to occur at 30,000 miles?

(g) Restore all the inputs to their original values. Arrange your spreadsheet so that it will print on one page, with the chart below the data table.

(If you're thinking of buying a car, remember that there's a lot more that goes into the cost of driving one car or another (or any car at all) than just these simple computations using initial cost and miles driven.)

(a) Questions about the conventional Camry.

(i) Once you own the car, how much does it cost to run, in dollars per mile? Does your answer make sense?
The cost in dollars per mile is

$$3.50\frac{\$}{\text{gallon}} \times \frac{1\text{ gallon}}{26\text{ miles}} = 0.1346\frac{\$}{\text{mile}}.$$

That's about 13 cents per mile, which makes sense to me.

(ii) Calculate the total cost (purchase plus gasoline) to drive the conventional Camry 10,000 miles.

$$\$19,720 + 0.1346\frac{\$}{\text{mile}} \times 10,000\text{ miles} = \$21,066.1538 \approx \$21,000.$$

(iii) Write the linear equation that computes the total cost C of driving the conventional Camry M miles if gasoline costs \$3.50/gallon.

$$C = 19,720 + 0.1346\,M.$$

(iv) Identify the slope and the intercept of this equation, with their units.
The slope is 0.1346 dollars per mile. The intercept is 19,720 dollars.

(b) Open the spreadsheet `ConventionalvsHybrid.xlsx`. Enter the data from the table on page 1 and the cost of gasoline in the appropriate cells. What formula should you enter in cell `C15` to check your answer to part (i) ?

```
=C9+C6*B15/C10
```

(c) Copy your formula to cells `B14:D29` to fill in the table. Indicate here where you put $ signs in to keep Excel from changing row and column references.

```
=C$9+$C$6*$B15/C$10
```

Plan	Phone cost	Monthly charge	Two year cost
Cricket Wireless	499.99	55	1,819.99
Virgin Mobile	649	30	1,369

Table 7.19. Comparing cell phone plans [R10]

Note that I have no `$` before two of the `C` references so this formula will do the right thing when I move it to column `D` for the hybrid.

Create a properly formatted and labelled chart in Excel showing how the cost of driving each car depends on the number of miles driven. Use your graph along with the table to answer the following questions.

See `../Answers/ConventionalvsHybridSolution.xlsx`.

(d) If you drive 120,000 miles will you recover in gas savings the extra initial cost of the hybrid? Write a complete sentence or two and use appropriate precision for the numbers you use to make your argument.

No. The total cost for driving the hybrid is about $39K. That's about $3,000 more than the total cost for driving the conventional Camry.

(e) When will you recover the extra initial cost in gas savings if the government (re)instates a $3,000 tax rebate for hybrid purchases?

The breakeven point is at about 120,000 miles. According to Excel the costs for the conventional and hybrid are $35,873.85 and $35,927.94 respectively. These are only about $50 apart, which is much too much precision for a prediction that far away.

(f) With the original initial costs, how much would the price of gasoline have to be in order for the breakeven point to occur at 30,000 miles?

By trial and error, changing the gasoline price in the spreadsheet, I found out that gas would have to cost about $23 per gallon for this to happen.

Exercise 7.8.12. [S][Section 7.5][Goal 7.3] [Goal 7.5] Contract or not?

Table 7.19 provides data that appeared in a story in *The Boston Globe* on June 14, 2012 headlined "Pay full price for iPhone, avoid contract".

(a) How much would it cost (in total) to buy the Cricket phone and use it for two months?

(b) Write an equation for the total cost to buy and use the Virgin Mobile phone for M months.

(c) Identify the slope and the intercept of your equation, with proper units for each.

(d) Create an Excel spreadsheet and use Excel formulas to complete a table like this:

Months	Cricket	Virgin
0		
1		
...		
24		

.

(e) Check that your spreadsheet produces the answers in the table for 24 months.

(f) Create a properly labelled and formatted chart displaying the data in your table.

(g) When (in terms of months of use) would it be better to choose the Cricket phone?

(h) Suppose the monthly charge for the Cricket phone was just \$45 while that for the Virgin phone increased to \$35/month. Answer the previous question with this new data.

(a) How much would it cost (in total) to buy the Cricket phone and use it for two months?

Cricket phone two months: \$609.99.

(b) Write an equation for the total cost to buy and use the Virgin Mobile phone for M months.

$$\text{Virgin Cost} = 650 + 30M.$$

(c) Identify the slope and the intercept of your equation, with proper units for each.

The slope is 30 dollars/month; the intercept is 650 dollars.

(d) Create an Excel spreadsheet and use Excel formulas to complete the table.

See `../Answers/cellphoneplansolution.xlsx`.

(e) Check that your spreadsheet produces the answers in the graphic for 24 months.

It does.

(f) Create a properly labelled and formatted chart displaying the data in your table.

See `../Answers/cellphoneplansolution.xlsx`.

(g) When (in terms of months of use) would it be better to choose the Cricket phone?

Cricket is cheaper up to 6 months. Then Virgin is the better buy.

(h) Suppose the monthly charge for the Cricket phone was just \$45 while that for the Virgin phone increased to \$35/month. Answer the previous question with this new data.

Change values in cells `C9` and `D9`. Then the graphs cross at 14 months.

Exercise 7.8.13. [S][Section 7.6][Goal 7.2] [Goal 7.1][Goal 7.7] Regenerative braking.

When you apply the brakes in a Toyota Prius the car uses some of the energy of the forward motion to recharge the battery. The dashboard displays a little car icon each time that recharging has collected 50 watt-hours.

(a) Estimate the energy equivalent of each icon in gallons of gasoline.

(b) Estimate the dollar value of that gasoline.

(c) Compare your estimate to the dollar value of 50 watt-hours of electricity in your house (what it would cost to keep a 100 watt bulb on for half an hour).

(d) Discuss the value of the display.

[See the back of the book for a hint.] For part (a) you will have to look up the conversion factors among various forms of energy in order to convert watt-hours to gallons of gasoline. Part (d) is about psychology, not quantitative reasoning. Your answer might begin "The display is valuable because ... On the other hand ..."

(a) Estimate the energy equivalent of each icon in gallons of gasoline.

 `www.calculateme.com/energy/kilowatt-hours/to-gallons-of-gas/` I discovered that 0.05 kilowatt-hour equals 0.0015 gallons of gasoline so each icon is equivalent to between one and two tenths of a percent of a gallon of gasoline.

(b) Estimate the dollar value of that energy.

 I'll use $3 per gallon for the price of gasoline. Then 0.0015 gallons will cost about 0.45 cents. Call it half a penny.

 That was a much smaller number than I expected.

(c) Compare your estimate to the dollar value of 50 watt-hours of electricity in your house (what it would cost to keep a 100 watt bulb on for half an hour).

 Electricity in my house costs about 10 cents per kilowatt-hour. So there too 50 watt-hours does indeed cost half a penny! So I think my answer to the previous part of the problem is probably right, even though it's surprising.

(d) Discuss the value of the display.

 Regenerative braking doesn't save much gasoline money at all — just pennies for even moderately long trips. So the value of the gas is negligible. But the display might be valuable. I know from driving my daughter's Prius on occasion and watching others drive that the display of the energy efficiency encourages people to drive more slowly and use less gas.

Exercise 7.8.14. [S][Section 7.6] [Goal 7.1][Goal 7.7] Computers don't sleep soundly.

The website `michaelbluejay.com/electricity/computers.html` gives information about how much energy a computer uses while asleep, in standby mode, or in use. The iMac G5, for example, uses 97 watts while "doing nothing", compared to 3.5 watts while asleep.

(a) What do you think "doing nothing" means?

(b) If this type of computer is doing nothing all day (24 hours), how much electricity does it use? Express your answer in kilowatt-hours.

(c) Now suppose the computer goes to sleep after 15 minutes of doing nothing. How much electricity does it use in an idle day?

(d) If a kilowatt-hour of electricity costs 20 cents, how much money is saved in a day because the computer is smart enough to go to sleep?

(a) What do you think "doing nothing" means?

According to the quote, "doing nothing" can't really be doing nothing, since it uses 97 watts. That's probably running the screen saver program, and checking from time to time for email or updates to various websites — things the computer does when it's turned on even if no one is typing at the keyboard or using the mouse.

(b) If this type of computer is doing nothing all day (24 hours), how much electricity does it use? Express your answer in kilowatthours.

Consuming electricity at a rate of 97 watts for a day uses 97 watt-days. That's $24 \times 97 = 2,328$ watt-hours or 2.3kwh of electricity.

(c) Now suppose the computer goes to sleep after 15 minutes of doing nothing. How much electricity does it use in an idle day?

A lot less: $0.25 \times 97 + 23.75 \times 3.5 = 107.375$ watt-hours, which is about 0.11 kwh.

(d) If a kilowatt-hour of electricity costs 20 cents, how much money is saved in a day because the computer is smart enough to go to sleep?

The sleep saves about 2.2 kwh of electricity, worth 44 cents.

Exercise 7.8.15. [S][C][Section 7.6] [Goal 7.1][Goal 7.7] How Much Water Does Pasta Really Need?

On February 24, 2009 *The New York Times* published an article by Harold McGee addressing that question.

McGee's kitchen experiments convinced him that he could cook pasta in far less water than is customary. Since (he says) we consume about a billion pounds of pasta a year:

> My rough figuring indicates an energy savings at the stove top of several trillion B.T.U.s. At the power plant, that would mean saving 250,000 to 500,000 barrels of oil, or $10 million to $20 million at current prices. Significant numbers, though these days they sound like small drops in a very large pot. [R11]

(a) Verify the author's conversion of "several trillion B.T.U.s" to barrels of oil and then to dollars.

(b) Does McGee's estimate of a billion pounds of pasta per year make sense?

(c) How much water do Americans use cooking pasta? How much would they use if they followed McGee's advice?

(d) Does not boiling the extra water really save the amount of energy McGee claims?

(a) Verify the author's conversion of "several trillion B.T.U.s" to barrels of oil and then to dollars.

The website `www.unitjuggler.com/convert-energy-from-boe-to-MMBtu.html` says that the energy in one barrel of oil is 5.55 million Btu. If I take "several trillion Btu" to be "2 trillion Btu" then that's equivalent to (2 trillion)/(5.5 million) = 360 thousand barrels of oil. That's just about what McGee says.

On October 24, 2014 oil cost about $80/barrel. 250,000 barrels would cost about $20 million. That's double McGee's estimate, so when he wrote in February 2009 oil must have cost about $40/barrel. The web site `www.macrotrends.net/1369/crude-oil-price-history-chart` shows a cost of about $42/barrel for January 2009.

(b) Does McGee's estimate of a billion pounds of pasta per year make sense?

That would be about

$$\frac{1 \text{ billion pounds}}{100 \text{ million households}} = 10\frac{\text{pounds}}{\text{household}}.$$

I guess I can believe that order of magnitude — it's pasta for dinner about once a month on average. Some households never cook it; those that do probably have it much more often than that.

(c) How much water do Americans use cooking pasta? How much would they use if they followed McGee's advice?

If the usual method is to use two quarts of water for a pound, this much pasta would take two billion quarts of water. McGee suggests two cups instead of two quarts. Since there are four quarts in a cup, doing pasta McGee's way would take only one fourth as much water, or half a billion quarts.

(d) Does not boiling the extra water really save the amount of energy McGee claims?

To answer this I looked up how much energy it takes to boil water. Several websites told me it was about 350 kilojoules per quart. Google told me that was about 330 Btu per quart. To boil the extra billion and a half quarts of water would take about 500 trillion Btu. That's more than the "several trillion" McGee claims. I wonder where I went wrong — if I did.

Exercise 7.8.16. [S][Section 7.6][Goal 7.7] Solar energy.

On May 1, 2013, *The Arizona Republic* reported on the $500 million Arlington Valley Solar Energy II project near Phoenix. The article said

> [the project] will have 127 megawatts of capacity when finished. One megawatt is enough electricity to supply about 250 Arizona homes at once, when the sun is shining on the solar panels. [R12]

(a) How many watts of power does the average Arizona home need to run its appliances when the sun is shining?

(b) Research the power requirements of several typical home appliances: air conditioners, stoves, television sets, ... Then decide whether the article's claim about home power requirement on a sunny day in Arizona is reasonable.

(a) How many watts of power does the average Arizona home need to run its appliances when the sun is shining?

Since one megawatt powers 250 homes, one home requires

$$\frac{1}{250}\text{ mw} = \frac{1000}{250}\text{ kw} = 4\text{ kw} =$$

(b) Research the power requirements of several typical home appliances: air conditioners, stoves, television sets, ... Then decide whether the article's claim about home power requirement on a sunny day in Arizona is reasonable.

The website `www.wholesalesolar.com/solar-information/how-to-save-energy/power-table` lists the power requirements for many devices. I found out there that central air conditioning can call for 5000 watts (1000 watts for room units). A TV might need 200w. Add in the lights and computers, a toaster or microwave or electric stove, electric water heat and other appliances and I think 4 kilowatts is probably a low estimate.

Exercise 7.8.17. [S][C][Section 7.6] [Goal 7.1][Goal 7.7] Solar power at Wellesley College.

The sign on a solar panel array at Wellesley College reads:

Solar Photovoltaic Array This 10-kilowatt Solar PV Array is composed of 48 panels, each 210 watts. It will generate approximately 13,000 kilowatt hours of electricity per year, enough to power 2 homes, 32 metal halide street lights or 85 LED street lights for an entire year. For real time electrical output please go to: `www.sunwatchmeter.com/home/day/wellesley-college` PLEASE KEEP OFF THE PANELS

You can see a picture at `www.theswellesleyreport.com/2010/09/wellesley-college-saves-the-planet/solar-panel/`

(a) Check the consistency of some of the numbers.

(b) How many hours of sunshine per day do the designers expect the installation to see?

(c) Visit the website on the sign and write about it. What do the graph and the meters represent? What is happening there now?

(a) Check the consistency of some of the numbers.

The total power is $48 \times 210 \approx 50 \times 200 = 10,000$ watts, just at the sign says. (The exact arithmetic comes to $48 \times 210 = 10,080$ watts.)

(b) How many hours of sunshine per day do the designers expect the installation to see?

13,000 kilowatt-hours per year is $13,000/365 \approx 36$ kilowatt-hours per day. The 10,000 kilowatt power supply would take 3.6 hours of sunshine to generate than much electricity.

Three and a half hours of sunshine a day sounds like a reasonable average.

(c) Visit the website on the sign and write about it. What do the graph and the meters represent? What is happening there now?

I visited The graph shows time of day on the x-axis and power (in watts) on the y-axis. The green area under that graph is the number of watt-hours of electric energy generated so far in the day. The first meter shows the power of the array at the moment. The meter on the right shows whether Wellesley is generating more or less electricity than it is using at the moment.

October 28, 2015, at 2 p.m. the power was 849 watts (less than a kilowatt) and Wellesley was getting electricity from the grid.

Exercise 7.8.18. [C][Section 7.6][Goal 7.7] Wind power.

From *The Los Angeles Times*, March 1, 2009:

> The U. S. last year surpassed Germany as the world's No. 1 wind-powered nation, with more than 25,000 megawatts in place. Wind could supply 20% of America's electricity needs by 2030, up from less than 1% now, according to a recent Energy Department report. [R13]

(a) What do these data say when you calculate wind power in megawatts per person, or as a percentage of the total power available?

(b) Explain why it might or might not be true to say that when this article appeared the U.S. now produced more wind energy than Germany?

Exercise 7.8.19. [S][Section 7.6][Goal 7.7] [Goal 7.1] World solar power.

In a posting on their website on July 31, 2013, the Earth Policy Institute reported that

> The world installed 31,100 megawatts of solar photovoltaics (PV) in 2012 — an all-time annual high that pushed global PV capacity above 100,000 megawatts. There is now enough PV operating to meet the household electricity needs of nearly 70 million people at the European level of use. [R14]

A graphic at the website `www.wec-indicators.enerdata.eu/household-electricity-use.html` shows that average household electricity consumption in Europe in 2013 was about 4,000 kwh/year. [R15]

(a) Use the data in this exercise to estimate the average number of hours per day that these solar panels are producing electricity.

(b) How does average household electricity consumption in the U.S. compare to that in Europe?

[See the back of the book for a hint.] It should not be hard to find a website that helps with the second question.

(a) Use the data in this exercise to estimate the average number of hours per day that these solar panels are producing electricity.

The quote says that 100,000 megawatts is power for 70 million people. If I estimate three people per household on average that's about 25 million households, which is 25 megahouseholds. That comes to about 4,000 watts, or 4 kilowatts per household.

Since European households use 4,000 kilowatt-hours per year, that solar panels would have to be working 1,000 hours per year to generate that electricity — about one eighth of the 8,000 hours per year. So the panels would be working about 3 hours per day.

(b) How does average household electricity consumption in the U.S. compare to that in Europe?

The same website that provided European household use shows shows a value of about 12,000 kwh/year for the United States. That's about three times the European average.

Exercise 7.8.20. [S][C][Section 7.6] [Goal 7.1][Goal 7.7] Chilling out by the quarry.

On August 16, 2010 *The Boston Globe* described a local business's plan to cool its corporate facility with water from a nearby quarry rather than with conventional air conditioning.

> [Director of facilities] Dondero estimated that the cooling system, which eliminates the need for any type of refrigerant in the building, saves about $75,000 a year, reduces annual water use by one million gallons, and cuts yearly energy use by about 300,000 kilowatt hours — enough to power about 30 homes. [R16]

(a) What rate in dollars per kwh is Dondero using to support his assertion that this change will save $75,000 a year?

(b) Is the claim that 300,000 kilowatt-hours would power 30 homes for a year reasonable?

(a) What rate in dollars per kwh is Dondero using to support his assertion that this change will save $75,000 a year?

This is easy if I just work with the units. I want dollars per kwh so I divide

$$\frac{\$75,000}{300,000\ \text{kwh}} = 0.25\frac{\$}{\text{kwh}}.$$

I didn't even need a calculator to find that nice round number.

(b) Is the claim that 300,000 kilowatt-hours would power 30 homes for a year reasonable?

There are lots of websites that begin to answer this question. I wanted one I thought might be authoritative, so I started at the U.S. Energy Information Administration FAQ at `www.eia.gov/tools/faqs/index.cfm`. There the answer to the question "How much electricity does an American home use?" is

> In 2008, the average annual electricity consumption for a U.S. residential utility customer was 11,040 kwh

The newspaper article assumes 10,000 kwh per household per year for 30 homes. That's the right order of magnitude.

Exercise 7.8.21. [S][Section 7.6][Goal 7.1] [Goal 7.7] Energy savings at MIT.

On March 26, 2011 Jon Coifman wrote in *The Boston Globe* that

> In just 36 months, [MIT and NStar] plan to cut the university's energy use 15 percent — enough to power 4,500 Massachusetts homes for a year. [R17]

(a) If all of MIT's energy use were devoted to powering Massachusetts homes, how many homes would that be?

(b) Compare your answer in part (a) to the number of homes in Cambridge.

(c) Estimate MIT's total annual energy use, in Btus.

(d) Convert your answer to the previous question from Btus to watt-hours.

(a) If all of MIT's energy use were devoted to powering Massachusetts homes, how many homes would that be?

It follows from the article that MIT uses enough energy in a year to power $4,500/0.15 = 30,000$ homes.

(b) Compare your answer in part (a) to the number of homes in Cambridge.

Cambridge has about 100,000 people, so about 30,000 homes. That means MIT uses as much energy as all the rest of the city (domestically).

Wikipedia says Cambridge has about 40,000 homes, but my estimate is the same order of magnitude, so the conclusion remains the same.

(c) Estimate MIT's total annual energy use, in Btus.

Wikipedia says annual household energy use in the Northeast is about 180,000,000 Btu, half of which is electricity (`en.wikipedia.org/wiki/File:US_household_energy_usage.png`).

$$\begin{aligned} &30,000 \text{ households} \times 180,000,000 \frac{\text{Btu}}{\text{household}} \\ &= 5,400,000,000,000 \text{ Btu} \\ &= 5.4 \text{ teraBtu.} \end{aligned}$$

(d) Convert your answer to the previous question from Btus to watt-hours. Write your answer using the appropriate metric prefix, not with lots of zeroes.

1 kilowatt-hour = 3412.1416 Btu, so

$$5.4 \times 10^{12} \text{ Btu} \times \frac{1 \text{ kilowatt-hour}}{3412.1416 \text{ Btu}} = 1.58 \text{ terawatt-hours.}$$

Exercise 7.8.22. [S][Section 7.6][Goal 7.1] [Goal 7.7] The Governor gets the units wrong.

On June 7, 2011 *The Norwich Bulletin* reported that

> Connecticut Governor Daniel Malloy recently signed off on a deal to tax electricity generators one quarter of one cent per kilowatt hour, or 25 cents per $100. [R18]

(a) What is wrong with the units in this quotation?

(b) Estimate the percentage change in the cost of electricity that would result from a one-quarter of one cent increase per kilowatt-hour.

(c) What do you think Governor Malloy intended to say?

[See the back of the book for a hint.] What did electricity cost in the spring of 2011?

(a) What is wrong with the units in this quotation?

The quote says that some number of cents per kilowatt-hour is equivalent to some number of cents per dollar. That makes no sense at all.

(b) Estimate the percentage change in the cost of electricity that would result from a one-quarter of one cent increase per kilowatt-hour.

In the spring of 2011 a kilowatt-hour costs about 10 cents in Massachusetts. I need to find out what percentage 0.25 cents (a quarter of a cent) is of 10 cents. That's easy: $0.25/10 = 0.025 = 2.5\%$.

(c) What do you think Governor Malloy intended to say?

I think the Governor meant to say that a quarter of a cent per kilowatt-hour is 25 cents per hundred kilowatt-hours, not per hundred dollars.

Exercise 7.8.23. Your total federal tax bill.
Moved to Extra Exercises at `commonsensemathematics.net` .

Exercise 7.8.24. [U][Section 7.7][Goal 7.1] [Goal 7.5][Goal 7.6] Your total federal tax bill.

Modify the graph in Figure **??** to show how total tax and the effective tax rate for (income tax + Social Security) depends on income.

Exercise 7.8.25. [S][C][Section 7.7][Goal 7.1] [Goal 7.6] President Obama's income tax.

According to the White House website

> [The President] and the First Lady filed their [2013] income tax returns jointly and reported adjusted gross income of $481,098. The Obamas paid $98,169 in total tax.
>
> The President and First Lady also reported donating $59,251 — or about 12.3 percent of their adjusted gross income — to 32 different charities. The largest reported gift to charity was $8,751 to the Fisher House Foundation. The President's effective federal income tax rate is 20.4 percent. . . . The President and First Lady also released their Illinois income tax return and reported paying $23,328 in state income tax. [R19]

The President itemized deductions, so he could deduct charitable contributions and state tax from his adjusted gross income:

> In the United States income tax system, adjusted gross income (AGI) is an individual's total gross income minus specific deductions. Taxable income is adjusted gross income minus allowances for personal exemptions and itemized deductions. [R20]

(a) With the information given, what is the largest possible value for the President's taxable income? What tax bracket would he be in?

(b) Use the Married Filing Jointly brackets and rates in the spreadsheet at `Federalindividualrat` to compute the Obamas' 2013 federal income tax bill for your answer to part (a). If your result does not match the reported figure, what might explain the difference?

(c) If the Obamas had not made those charitable contributions the money would be part of their taxable income. Use your answers to the previous questions to answer these.

   (a) What would their taxable income have been? What bracket would that have put them in? What would their tax have been?
   (b) What fraction of the contribution was (essentially) made by the government?
   (c) What fraction of the Obamas' income did they contribute to charity?
   (d) Did they tithe?
   (e) How does their contribution compare to the national average?

You can find the arithmetic for my solution in the spreadshee `../Answers/PresidentsTaxSoluti`

(a) With the information given, what is the largest possible value for the President's taxable income? What tax bracket would he be in?

By subtracting the Illinois state tax and charitable contributions from the adjusted gross income I found a taxable income of $398,519. That's just barely into the 35% bracket.

(b) Use the Married Filing Jointly brackets and rates in the spreadsheet at `Federalindividualrat` to compute the Obamas' 2013 federal income tax bill for your answer to part (a). If your result does not match the reported figure, what might explain the difference?

On that taxable income the tax would be $107,828. That's about $9,000 more than what the Obamas paid. There were probably other deductions we don't know about.

(c) If the Obamas had not made those charitable contributions the money would be part of their taxable income. Use your answers to the previous questions to answer these.

(a) What would their taxable income have been? What bracket would that have put them in? What would their tax have been?
Their taxable income would have been $457,770, putting them in the top (39.6%) bracket. They would have paid $125,846 in tax.

(b) What fraction of the contribution was (essentially) made by the government?
The government would have collected $18,018 more in tax if the Obamas had not donated $59,251 to charity. That means the government "donated" $\$18,018/\$59,251 = 0.30 = 30\%$ of the Obamas' contribution.

(c) What fraction of the Obamas' income did they contribute to charity?
12.3 percent, as the quote says. I checked the arithmetic.

(d) Did they tithe?
Yes — it's more than ten percent.

(e) How does their contribution compare to the national average?
The website `www.philanthropy.com/article/The-Stubborn-2-Giving-Rate/154691` reported that

> Donations from individuals — almost $229-billion [in 2012] . . . generally hover at around 2 percent of disposable income, with occasional blips up or down, according to the Giving USA Foundation, which helps compile the annual philanthropy report.

So the Obama's donated about six times as much, as a proportion of their income.

Exercise 7.8.26. [S][Section 7.7][Goal 7.1][Goal 7.6] Using the tax table.

Use Table 7.11 to answer the following questions.

(a) Compute the tax due in 2019 on a net taxable income of $80K. Show your work.

(b) Compute the effective tax rate for that income.

(c) Check your answers with those in the spreadsheet `GraduatedTax.xlsx`.

(a) The tax on an income of $80K is

$$10\% \times \$9700 + 12\% \times (\$39475 - \$9700) + 22\% \times (\$80000 - \$39475) = \$13,458.5016.82$$

(b) The effective tax rate is $\$13,458.50/\$80000 = 16.82\%$.

(c) The answers match those I get by entering $80,000 in cell `G7`.

Exercise 7.8.27. [S][Section 7.7][Goal 7.1][Goal 7.6] Taxes and inflation.

The spreadsheet `Federalindividualratehistory.xlsx` contains a complete history of income tax brackets and rates from the inception of the income tax in 1913 through 2013.

(a) Compute the tax due in 2003 for a single taxpayer with a net taxable income of $30K. What is her effective tax rate?

(b) Suppose that taxpayer received raises each year that kept up with inflation. Use an inflation calculator to calculate her net taxable income in 2013.

(c) Use the 2013 tax tables to compute her tax in 2013. What is her effective tax rate?

(d) Compare her 2003 tax and effective tax rate with her 2013 tax and effective tax rate, taking inflation into account. Has her tax gone up, or down, or stayed the same?

(a) The tax in 2003 on an income of $30K was

$$10\% \times \$7000 + 15\% \times (\$28400 - \$7000) + 25\% \times (\$30000 - \$28400) = \$4310.$$

Her effective tax rate was $\$4310/\$30000 = 0.143666667 \approx 14.46\%$.

(b) The inflation calculator at `data.bls.gov/cgi-bin/cpicalc.pl` says that $30,000 in 2003 has the same buying power as $37,982.12 in 2013

(c) The tax in 2013 on an income of $37,982 is

$$\begin{aligned} 10\% \times \$8925 + 15\% \times (\$36,250 - \$8925) \\ + 25\% \times (\$37,982 - \$36,250) = \$5424. \end{aligned}$$

Her effective tax rate is $\$5424/\$37,982 = 0.142811 \approx 14.28\%$.

(d) Her tax has gone up by $1114, or 25.8%. Her income has gone up by $\$37982/\$30000 = 1.266$, which is a 26.6% increase. That's a little bit more than the percentage increase in her taxes, which is why her effective tax rate decreased by just 0.18 percentage points. That's not a lot. The IRS did a pretty good job adjusting the brackets for inflation.

Exercise 7.8.28. [S][Section 7.3][Goal 7.2] [Goal 7.3] Pandora growing fast!

On June 10, 2011, CNN Money reported that the internet music site Pandora is adding new users at the rate of one per second. Between February and April the number of users grew from 80 to 90 million. [R21]

Is the slope of one user per second correct based on the February and April numbers of users?

Is the slope of one user per second correct based on the February and April numbers of users?

For a quick and dirty estimate I'll use 100 days for February, March and April. Then I asked Google and was told

$$\boxed{\text{(10 million miles) per (100 days)} = 1.15740741 \text{ miles per second}}\ .$$

So the estimate of one user per second is just about right.

Exercise 7.8.29. [S][Section 7.6][Goal 7.7] Bicycle power in Times Square.

On December 30, 2012 *The Boston Globe* reported on an Associated Press story about six bicycles that would help illuminate the famed falling ball in Times Square on New Year's Eve.

> Each bike will generate an average of 75 watts an hour. It takes 50,000 watts to light up the ball's LEDs. [R22]

Unfortunately, the Associated Press reporter is quite confused about the difference between energy and power. The "generate . . . 75 watts an hour" in the quote makes no sense. We think what he or she is trying to say is that while someone is actually pedaling it each bike could power a 75 watt light bulb. All six bikes together could light up just 450 watts worth of LEDs.

(a) How many bikes would have to be pedaled simultaneously to light up all the ball's LEDs?

(b) Since there are only six bikes, people pedaling during the day will store the energy they generate in batteries, which will then be used to light the ball. Suppose the lights need to be on for two minutes while the ball drops at midnight.

How many hours of pedaling will it take to generate (and save) the electrical energy needed?

(a) How many bikes would have to be pedaled simultaneously to light up all the ball's LEDs?

Since each bike generates 75 watts (when it's being pedaled) and you need 50,000 watts to light the ball, you'd need $50,000/75 = 667$ bikes (rounding up).

(b) Since there are only six bikes, people pedaling during the day will store the energy they generate in batteries, which will then be used to light the ball. Suppose the lights need to be on for two minutes while the ball drops at midnight.

How many hours of pedaling will it take to generate (and save) the electrical energy needed?

To light 50,000 watts worth of LEDs for two minutes requires 100,000 watt-minutes of electricity. An hour of pedaling on one bikes will generate 75 watt-hours of electricity. That's $75 \times 60 = 4,500$ watt-minutes. You would need $100,000/4,500 = 22.22$ hours of pedaling to generate the electricity you need. That's about four hours for six bikes.

I'm surprised that the number is so small.

I can check this answer by using my answer to the first part. There I figured out that 667 bikes simultaneously would do the job. To keep the lights on for two minutes I'd need $2 \times 667 = 1334$ bike-minutes of power. That's $1334/60 = 22.23$ bike-hours — the same answer (off in the second decimal place because I rounded up).

Exercise 7.8.30. [S][Section 7.3][Goal 7.2][Goal 7.3] Flying twice as far.

A curious traveler asked this question on stack exchange:

> A flight from Los Angeles to Albuquerque is about 2 hours but is ≈ 670.2 miles.
>
> A flight from San Jose to Chicago is 4 hours but is ≈ 1859.0 miles.
>
> Can anyone explain why the travel time from San Jose to Chicago is not longer and closer to 5.75 hours?
>
> If the distance increase by 2, shouldn't the time increase by a factor of 2 as well? [R23]

(a) Write a linear model for this question. Takeoff and landing will take a fixed amount of time. Actual travel in the air will take time proportional to the distance traveled. Think about which of the variables (time and distance) is the independent variable, and identify the slope and intercept with their units.

(b) Use the data in the quotation to estimate the two constants in your linear model.

(c) Compare your answer to those at the link to the quotation.

(a) Write a linear model for this question. Takeoff and landing will take a fixed amount of time. Actual travel in the air will take time proportional to the distance traveled. Think about which of the variables (time and distance) is the independent variable, and identify the slope and intercept with their units.

In this problem the dependent variable y is the travel time, in hours. The independent variable x is the distance flown, in miles. The intercept b is the time in minutes for takeoff and landing. The slope is m. Its units are minutes per mile. That's the tricky part of this question — we're used to thinking of speed in units time/distance, not distance/time.

The linear model is then the familiar $y = mx + b$.

(b) Use the data in the quotation to estimate the two constants in your linear model.

I know from the data (after a little rounding) that

$$4 = m \times 1860 + b$$

and

$$2 = m \times 670 + b.$$

Subtracting tells me that

$$2 = (1860 - 670)m = 1190m \approx 1200m$$

so m is about (2/1200) hours per mile, or 600 miles per hour. That's a pretty good approximation for airline cruising speed. Putting $m = 1/600$ into the first equation tells me that

$$4 = 1860/600 + b \approx 3 + b$$

so $b \approx 1$ hour — half an hour each for takeoff and landing.

(c) Compare your answer to those at the link to the quotation.

The link says it's reasonable to estimate the cruising speed at about 500 miles per hour and the takeoff and landing time at half an hour. Those numbers fit the data too.

Exercise 7.8.31. [S][Section 7.3][Goal 7.2][Goal 7.3] Express lane?

In *The Boston Globe* on November 27, 2015 you could read that

> Amid the holiday grocery shopping madness, every line feels like the wrong one. And yet, some are wronger than others. Given equally capable cashiers, you are often better off bypassing the express lane. Research conducted at a large, unnamed, California grocery store found that while each item adds 3 seconds to the check-out time, it takes 41 seconds for a person to move through the line even before their items are added to the tally. Bottom line: The big time-consumers are not the items, but the small talk

> and the paying, says Dan Meyer, who has a doctorate in math education from Stanford University. [R24]

Suppose you have 10 items in your cart, so you are allowed to use the express lane. How much longer must the line there be (compared to the regular lane) to make the wait in the regular lane less?

You can answer that question with any strategy that makes sense to you, as long as you explain what you're thinking. If you need a starting place, one way is to use these steps:

(a) Write the linear equations showing how the time it takes a shopper to check out depends on the number of items in her cart. What are the slope and intercept, with their units?

(b) Suppose shoppers in the express lane buy 6 items (on average), while those in the regular lane buy about 20. Write the linear equations showing your waiting time in each line depends on the number of shoppers ahead of you.

(c) Now work on the main question — which line should you join when you have 10 items in your cart? How much longer must the express lane line be to make the wait on the regular lane line less?

[See the back of the book for a hint.] The answer depends on the relative lengths of the lines, not on the absolute difference in the lengths.

(a) Write the linear equations showing how the time it takes a shopper to check out depends on the number of items in her cart. What are the slope and intercept, with their units?

Let T be the time (in seconds) and I the number of items in the cart. Then the equation for checkout time is

$$T = 3I + 41.$$

The slope is 3 seconds/item and the intercept is 41 seconds.

(b) Suppose shoppers in the express line buy 6 items (on average), while those in the regular line buy about 20. Write the linear equations showing your waiting time in each line depends on the number of shoppers ahead of you.

Using my answer to (a) I see that I will have to wait 3*6 + 41 = 59 seconds for each customer ahead of me in the express line, so

$$W = 59C,$$

where W is the waiting time in seconds and C is the number of customers ahead of me. The slope is 59 seconds per customer. The intercept is 0 since if I'm first in line there's no wait.

The equation for the regular line has slope 3*20 + 41 = 101 seconds per customer:

$$W = 101C.$$

(c) Now work on the main question — which line should you join when you have 10 items in your cart? How much longer must the express line be to make the wait on the regular line less?

My first thought is that the single seconds in 59 and 101 are a distraction, and I will approximate them by 60 and 100 seconds. That means the wait behind 5 customers on the regular line is the same as the wait behind 3 customers on the express line — 300 seconds (5 minutes). To decide which line to join I would mentally count the customers in groups of 5 in the express line and groups of 3 in the regular line.

With arithmetic: if the number of customers on the express line is more than 5/3 = 1.7 times the number of customers on the regular line, it's better to wait on the regular line.

I can reach the same answer with algebra. Let R be the humber of customers on the regular line and E the number on the express line. Then I want the regular line when

$$61E > 101R$$

which is the same as

$$E > \frac{101R}{61} = \frac{101}{81}R \approx 1.7.$$

Review exercises.

Exercise 7.8.32. [A] If you drive at a rate of 50 miles per hour for 3 hours, how far have you driven? Identify each piece of this proportion: the quantities being measured and the proportionality constant, with the appropriate units.

Exercise 7.8.33. [A] You may remember from geometry that the circumference of a circle is directly proportional to the diameter of that circle. The relationship is

$$c = \pi d,$$

where c represents the circumference and d represents the diameter and $\pi \approx 3.14$ is the proportionality constant. If the diameter of a circle is doubled, how does the circumference change?

Exercise 7.8.34. [A] The cost of potatoes is proportional to the weight (in pounds) you buy. If potatoes cost $0.69 per pound, what is the cost for 3 pounds of potatoes?

Exercise 7.8.35. [A] The conversion from £ to U.S. $ is 1.53 $/£. How much is £200 worth in U.S.$?

Exercise 7.8.36. [A] Suppose that y is directly proportional to x. When $x = 16$, then $y = 4$.

(a) What is the proportionality constant?

(b) If $x = 32$, what is y?

(c) If $y = 32$, what is x?

Exercise 7.8.37. [A] Identify the slope and intercept in each of the following. When appropriate, state the units.

(a) $y = 2.5x + 6$.

(b) $y = -5x + 20$.

(c) $y = 300 + 40x$.

(d) $Q = 0.004E - 300$.

(e) He earns $9.25 per hour.

(f) To rent a car for one day, the cost is $25 plus $0.15 per mile.

(g) My new phone cost $25, plus a monthly charge of $15.

(h) The conversion from £ to U.S. $ is 1.53 $/£.

(i) The salesperson worked only on commission, earning 20% of the total amount sold.

Exercise 7.8.38. [A] Solve each problem.

(a) If $y = 2.5x + 6$ and $x = 4$, what is y?

(b) If $y = -5x + 20$ and $y = 0$, what is x?

(c) If $y = 300 + 40x$ and $x = -10$, what is y?

(d) If his salary is $9.25 per hour and he works 5 hours, how much does he earn?

(e) If the conversion from U.S. dollars to pounds sterling is 1.80 $/£, how much money would you get by changing $100 to £?

(f) If my new phone cost $25, and I pay a monthly charge of $15, what is my total cost after 10 months? When does my total cost reach $250?

Section A—Use if your filing status is **Single.** Complete the row below that applies to you.

Taxable income. If line 10 is—	**(a)** Enter the amount from line 10	**(b)** Multiplication amount	**(c)** Multiply (a) by (b)	**(d)** Subtraction amount	**Tax.** Subtract (d) from (c). Enter the result here and on the entry space on line 11a.
At least $100,000 but not over $157,500	$	× 24% (0.24)	$	$ 5,710.50	$
Over $157,500 but not over $200,000	$	× 32% (0.32)	$	$ 18,310.50	$
Over $200,000 but not over $500,000	$	× 35% (0.35)	$	$ 24,310.50	$
Over $500.000	$	× 37% (0.37)	$	$ 34,310.50	$

Figure 7.20. Tax calculation (2018) [R25]

Exercises added for the second edition.

Exercise 7.8.39. [U][S][Section 7.7][Goal 7.2][Goal 7.6] Instructions from the IRS.

Figure 7.20 is the form the Internal Revenue Service provides for computing 2018 tax on incomes over $100,000.

(a) Calculate the tax on an income of $150,000 using this form. (That income is the amount on line 10.)

(b) Calculate the tax on an income of $150,000 using the procedure in Figure 7.17. Check that you get the same answer.

(c) Explain how the IRS arrived at the figure $5,710.50 in column (d) in the first row of the form.

(a) Calculate the tax on an income of $150,000 using this form. (That income is the amount on line 10.)

The Google calculator tells me

$$\boxed{150{,}000 * 0.24 - 5{,}710.50 = 30289.5}$$

so the tax is $30,289.50.

(b) Calculate the tax on an income of $150,000 using the procedure in Figure 7.17. Check that you get the same answer.

The Google calculator tells me

$$\boxed{(150{,}000 - 82{,}500) * 0.24 + 14{,}089.50 = 30289.5}$$

which is the same answer as in (a).

(c) Explain how the IRS arrived at the figure $5,710.50 in column (d) in the first row of the form.

The instructions ask you to calculate the tax as if all the income was at your highest bracket, and then subtract an amount that corrects for the fact that some of your income should be taxed at a lower rate.

Exercise 7.8.40. [U][C][N] Does virtual save energy?

> MOST PEOPLE take for granted the Earth-friendly nature of electronic communication. Paperless, ink-free, no shipping supplies, no gas for transportation: the environmental benefits of virtual communication are obvious. But the reality is more complicated, at least according to a growing number of concerned technology experts and scientists. Vast stockpiles of digital data waste energy, too.
>
> Everyday emails aren't to blame. But large photo and video attachments, cluttered inboxes, and massive email forwards may be. Some analysts estimate that emailing a 4.7-megabyte attachment — the equivalent of four large digital photos — can use as much energy as it takes to boil about 17 kettles of water. The problem is magnified when large emails are forwarded to many people and left in inboxes undeleted. As long as emails remain in your inbox, the data they create is physically stored somewhere.
>
> And that's where the problems arise: The total amount of digital storage worldwide is approaching 1 zettabyte, or 1 million times the contents of the Earth's largest library. Currently, that information is archived on equipment with a mass equivalent to 20 percent of Manhattan. Global data storage is expected to reach 35 zettabytes by 2020, which means more equipment, land, and energy. The information industry already accounts for approximately 2 percent of global carbon dioxide emissions. That's the same amount as the airline industry blasts into the atmosphere. Coupled with the rapid increase in stored data, it's an unsustainable scenario.
>
> Technology firms must create systems that store data with less energy, and governments should provide incentives for them to do so. Just as important, consumers must demand products that save energy, and use websites like Flickr and MediaFire that allow them to share large files without emailing. Better still, they could consider keeping some of those embarrassing photos and home videos to themselves. [R26]

Exercise 7.8.41. [U][N] Green gas?

Robert Bryce's op-ed in *The New York Times* on June 7, 2011 (`www.nytimes.com/2011/06/08/opinion/08bryce.html`) has lots of interesting numbers about the costs in steel and land area for solar and wind electricity generation.

Exercise 7.8.42. [U][N] Every little bit counts.

On March 5, 2012 *The Boston Globe* reported on the Ocean Renewable Power Company's plans to install tidal powered generators in Maine:

> The first unit capable of powering 20 to 25 homes will be hooked up to the grid this summer, and four more units will be installed next year at a total cost of $21 million …
>
> Eventually, Ocean Renewable hopes to install more units to bring its electrical output to 4 megawatts. [R27]

Exercise 7.8.43. [S][A][Section 7.5][Goal 7.2][Goal 7.3] Compact fluorescent bulbs.

Consumers are being encouraged to replace ordinary light bulbs with compact fluorescent bulbs. (CFLs). Soon they will be required to.

(This exercise should be updated to discuss LED bulbs too, and to use real rather than invented numbers.)

A CFL uses less energy than an ordinary incandescent bulb that produces the same amount of light, but it costs more to buy. This table provides data with which you can compare the two.

bulb	initial cost	power
ordinary	$2.00	100 watts
CFL	$9.00	25 watts

Suppose electricity costs $0.20 per kwh.

You can use pencil and paper, a calculator or Excel to do the arithmetic.

(a) Write a linear equation with which you can calculate the total cost C of using the ordinary bulb for H hours.

(b) What is the slope of that equation (with its units)?

(c) What is the intercept of that equation (with its units)?

(d) How much would it cost to buy the ordinary bulb and use it for 1000 hours?

(e) Write a linear equation with which you can calculate the total cost C of using the CFL for H hours.

(f) How much would it cost to buy the CFL and use it for 1000 hours?

(g) How long would you have to use the CFL to make it worth having paid the higher purchase price?

Are the five numbers given in this exercise reasonable?

What does “incandescent” mean? Why are incandescent light bulbs called that?

(a) Write a linear equation with which you can calculate the total cost C of using the ordinary bulb for H hours.

$$C = 2.00 + 0.1 \times 0.20H.$$

(b) What is the slope of that equation (with its units)?

The slope is 0.02 $/hour, or 2 cents/hour.

(c) What is the intercept of that equation (with its units)?

The intercept is $2.00.

(d) How much would it cost to buy the ordinary bulb and use it for 1000 hours?

$$C = \$2.00 + 0.02\frac{\$}{\text{hour}} \times 1000 \text{ hours} = \$22.00.$$

(e) Write a linear equation with which you can calculate the total cost C of using the CFL for H hours.

$$C = 9.00 + 0.025 \times 0.20H.$$

(f) How much would it cost to buy the CFL and use it for 1000 hours?

$$C = \$9.00 + 0.005\frac{\$}{\text{hour}} \times 1000 \text{ hours} \;= \$14.00.$$

(g) How long would you have to use the CFL to make it worth having paid the higher purchase price?

From my previous work, the answer will be less than 1000 hours. I can find the exact time by algebra or by trial and error. It turns out to be 467 or about 500 hours. That makes sense — the CFL costs a penny and a half less per hour to run, which means it will take about 500 hours to save the $7 difference in initial cost.

(h) Are the five numbers given in this exercise reasonable?

Waiting for student input.

(i) What does "incandescent" mean? Why are incandescent light bulbs called that?

"Incandescent" means "giving off light because it's hot". That's just how ordinary old fashioned light bulbs work. There's a thin filament (wire) inside that heats up and glows.

Exercise 7.8.44. [U][S][Section 7.6][Goal 7.7] Not flying to London.

In *The New York Times* on April 25, 2011 you could read that

> You can save so much energy by not flying to London [to collaborate with a coworker] that it will run a rack of computers for a year. [R28]

Estimate the energy costs of flying to London and running a rack of computers for a year to see if they are of the same order of magnitude.

If you're a physicist you can make these estimates with your common knowledge. If you're not, you can put together reliable information from the web. Try searching for the energy cost of flying an airplane and the energy cost of running a computer.

From `www.inference.phy.cam.ac.uk/withouthotair/c5/page_35.shtml` :

> Imagine that you make one intercontinental trip per year by plane. How much energy does that cost?
>
> A Boeing 747-400 with 240,000 litres of fuel carries 416 passengers about 8,800 miles (14,000 km). And fuel's calorific value is 10 kwh per litre. (We learned that in Chapter 3.) So the energy cost of one full-distance roundtrip on such a plane, if divided equally among the passengers, is
>
> $$\frac{2 \times 240{,}000 \text{ litre}}{416 \text{ passengers}} \times 10 \text{ kwh/litre} \approx 12{,}000 \text{ kwh per passenger.}$$
>
> If you make one such trip per year, then your average energy consumption per day is
>
> $$\frac{12{,}000 \text{ kwh}}{365 \text{ days}} \approx 33 \text{ kwh/day.}$$

The round trip airline distance from Boston to London is about 6,500 miles, so that trip will cost somewhat less than the trip above, figure 24 kwh/day.

At (say) 100 watts to run a computer (that's the right order of magnitude) you use about 2,400 watt-hours or 2.4 kwh in a day. So 24 kwh will power ten computers for a day. That's a pretty small rack, but the order of magnitude is right.

Exercise 7.8.45. [S] Express lane?

In *The Boston Globe* on November 27, 2015 you could read that

> Amid the holiday grocery shopping madness, every line feels like the wrong one. And yet, some are wronger than others. Given equally capable cashiers, you are often better off bypassing the express lane. Research conducted at a large, unnamed, California grocery store found that while each item adds 3 seconds to the

> check-out time, it takes 41 seconds for a person to move through the line even before their items are added to the tally. Bottom line: The big time-consumers are not the items, but the small talk and the paying, says Dan Meyer, who has a doctorate in math education from Stanford University. [R29]

Suppose you have 10 items in your cart, so you are allowed to use the express lane. How much longer must the line there be (compared to the regular lane) to make the wait in the regular lane less?

Hint: The answer depends one the relative lengths of the lines, not on the absolute difference in the lengths.

You can answer that question with any strategy that makes sense to you, as long as you explain what you're thinking. If you need a starting place, one way is to follow these steps:

(a) Write the linear equation showing how the time it takes a shopper to check out depends on the number of items in her cart. What are the slope and intercept, with their units?

(b) Suppose shoppers in the express lane buy 6 items (on average), while those in the regular lane buy about 20. Write a linear equation for each line showing your waiting time depends on the number of shoppers ahead of you.

(c) Now work on the main question — which line should you join when you have 10 items in your cart? How much longer must the express lane line be to make the wait on the regular lane line less?

(a) Write the linear equations showing how the time it takes a shopper to check out depends on the number of items in her cart. What are the slope and intercept, with their units?

Let T be the time (in seconds) and I the number of items in the cart. Then the equation for checkout time is

$$T = 3I + 41.$$

The slope is 3 seconds/item and the intercept is 41 seconds.

(b) Suppose shoppers in the express line buy 6 items (on average), while those in the regular line buy about 20. Write the linear equations showing your waiting time in each line depends on the number of shoppers ahead of you.

Using my answer to (a) I see that I will have to wait 3*6 + 41 = 59 seconds for each customer ahead of me in the express line, so

$$W = 59C,$$

where W is the waiting time in seconds and C is the number of customers ahead of me. The slope is 59 seconds per customer. The intercept is 0 since if I'm first in line there's no wait.

The equation for the regular line has slope 3*20 + 41 = 101 seconds per customer:

$$W = 101C.$$

(c) Now work on the main question — which line should you join when you have 10 items in your cart? How much longer must the express line be to make the wait on the regular line less?

My first thought is that the single seconds in 59 and 101 are a distraction, and I will approximate them by 60 and 100 seconds. That means the wait behind 5 customers on the regular line is the same as the wait behind 3 customers on the express line — 300 seconds (5 minutes). To decide which line to join I would mentally count the customers in groups of 5 in the express line and groups of 3 in the regular line.

With arithmetic: if the number of customers on the express line is more than 5/3 = 1.7 times the number of customers on the regular line, it's better to wait on the regular line.

I can reach the same answer with algebra. Let R be the humber of customers on the regular line and E the number on the express line. Then I want the regular line when

$$61E > 101R$$

which is the same as

$$E > \frac{101R}{61} = \frac{101}{81}R \approx 1.7R.$$

Exercise 7.8.46. [U] LED lightbulb.

Figure 7.21 shows the specifications for an LED bulb that costs \$11.00. It's meant to replace a 60 watt incandescent bulb that costs about a dollar.

(a) Check the claimed Estimated Yearly Energy Cost.

(b) Check that the Rated Life agrees with the claimed 22.8 year Life.

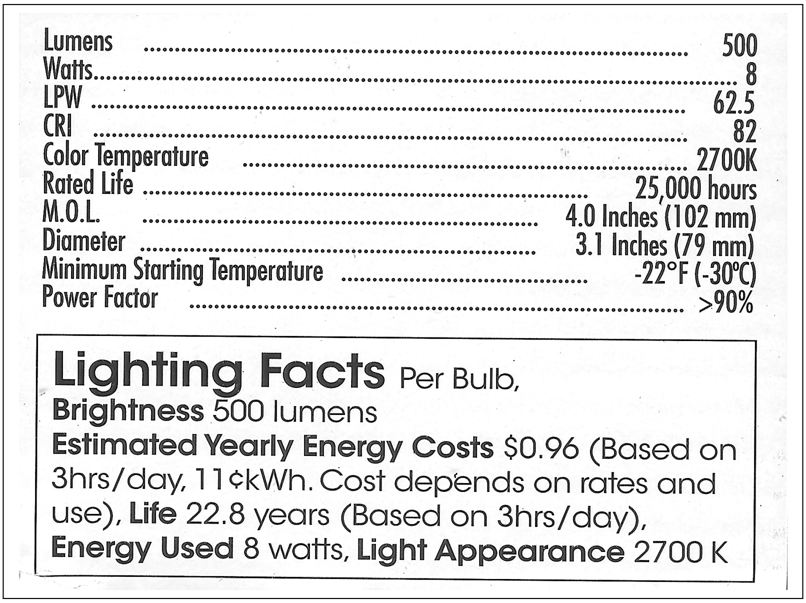
(c) How long will it take for the LED bulb to be cheaper overall than the incandescent bulb (counting both the initial cost and the cost of the electricity to run it) ?

(d) What does the acronym "LPW" stand for? Has it been computed correctly?

(e) What do the words "LED" and "incandescent" mean?

Exercise 7.8.47. [S] Kilowatt hours per day.

Professor Sir David Mackay's obituary in *The Telegraph* observed that



Lumens	500
Watts	8
LPW	62.5
CRI	82
Color Temperature	2700K
Rated Life	25,000 hours
M.O.L.	4.0 Inches (102 mm)
Diameter	3.1 Inches (79 mm)
Minimum Starting Temperature	-22°F (-30°C)
Power Factor	>90%

Lighting Facts Per Bulb,
Brightness 500 lumens
Estimated Yearly Energy Costs $0.96 (Based on 3hrs/day, 11¢kWh. Cost depends on rates and use), **Life** 22.8 years (Based on 3hrs/day), **Energy Used** 8 watts, **Light Appearance** 2700 K

Figure 7.21. Light bulb specs

> [His] genius was to express all forms of power consumption and production in a single unit of measurement — kilowatt hours per day (kWh/d). A 40 watt lightbulb, kept switched on all the time, uses one kWh/d, while driving the average car 50km a day consumes 40 kWh/d. [R30]

(a) Confirm that keeping a 40 watt lightbulb left switched on all day would use 1 kilowatt hour of energy.

(b) According to MacKay, "The amount of energy saved by switching off the phone charger [for a day] is exactly the same as the energy used by driving an average car for one second." How many people would have to switch of their cell phone chargers in a day to save as much energy as driving a car 50 km?

(c) Mackay claims that switching your cell phone charger off for one year saves as much energy as is needed for one hot bath. How many kilowatt hours per day does it take to heat that bath water? water?

(d) What did MacKay mean when he stated that gestures like turning of your cell phone charger were akin to "bailing out the Titanic with a teaspoon"? Do you agree?

(a) Confirm that keeping a 40 watt lightbulb left switched on all day would use 1 kilowatt hour of energy.

$$40 \text{ watts} \times 24 \text{ hours} = 960 \text{ watt-hours} \approx 1 \text{ kilowatt-hour}.$$

(b) According to MacKay, "The amount of energy saved by switching off the phone charger [for a day] is exactly the same as the energy used by driving an average car for one second." How many people would have to switch of their cell phone chargers in a day to save as much energy as driving a car 50 km?

I know from the quote that it takes one kilowatt-hour to drive a car 50 km. To figure out how much energy it takes to drive it for one second I need to estimate the speed of the car. To make the arithmetic easy I'll assume it's driving 50 km/hour (about 30 miles/hour). So the car needs 1 kilowatt-hour to drive for an hour.

MacKay says that each phone charger's energy savings could drive the car for a second, so it would take turning about 3,600 of them off for a day to save the energy to drive the car 50 km.

(c) MacKay claims that switching your cell phone charger off for one year saves as much energy as is needed for one hot bath. How many kilowatt-hours of energy does it take to heat that bath water?

Switching off the cell phone charger for a day saves 1/3,600 of a kilowatt-hour. Multiplying by (approximately) 360 days per year leads to a savings of 0.1 kilowatt-hours. So that's how much energy it takes to heat the bath water.

(d) What did MacKay mean when he stated that gestures like turning of your cell phone charger were akin to "bailing out the Titanic with a teaspoon"? Do you agree?

He means that the gesture is just a gesture, and won't make any difference when it comes to solving the real problem — saving the titanic or conserving energy. I agree with the conclusion. I do wonder if he thought about this quantitatively — how many people with teaspoons would it take to bail out the Titanic?

Exercise 7.8.48. [N] The Rosenfeld.

From *The New York Times* January 27 obituary of physicist Arthur Rosenfeld:

> [In 2010] a group of scientists proposed a unit of measurement in his name. The "Rosenfeld," they said, should refer to annual electricity savings of three billion kilowatt-hours — enough to eliminate the need for a coal plant. [R31]

Exercise 7.8.49. [U][N] Fuel economy in square meters?

`physics.stackexchange.com/questions/325733/why-can-fuel-economy-be-measured-in-square-meters`

> The fact that the gallon (or liter) is a volume is really irrelevant. What we're really measuring is distance per energy unit. It's just

> handy that all kinds of gasoline have pretty close to the same energy content for a given volume (about 32.8 kWh/gal, per Google), and technology/commerce makes it convenient to measure & sell liquids by volume. (But other liquid fuels, say ethanol or diesel, have kWh/volume, so we get different mpg from them.) So plug in the kWh/gal figure into your math, and get miles/kWh :-)

And of course

`what-if.xkcd.com/11/`

Exercise 7.8.50. [N][W] Home solar power. From *The Boston Globe* on May 28, 2017:

> The median price per watt of capacity for systems financed through the Mass Solar Loan Program is $4, and the median installation size is 8.1 kilowatts. Those numbers suggest a new solar system could easily run more than $30,000.
>
> Fortunately, several programs can help defray the costs. Federal tax incentives allow you to take a credit worth 30 percent of the cost of the installation. Massachusetts also offers a tax credit of 15 percent of the remaining cost after the federal incentive has been subtracted, with a maximum value of $1,000.
>
> Together, these credits could reduce the cost of a $30,000 system to roughly $20,000 — still a pretty hefty total. [R32]

Exercise 7.8.51. [U][N] What does a butcoin cost?

On January 22, 2018 you could read in *The New York Times* that

> . . . the computer power needed to create each digital token consumes at least as much electricity as the average American household burns through in two years . . . [R33]

There are more question opportunities later in the article.

Exercise 7.8.52. [U][N] What is fair?

In March 2018 *The New York Times* published an opinion piece headlined "A Billionaire and a Nurse Shouldn't Pay the Same Fine for Speeding"

> If Mark Zuckerberg and a janitor who works at Facebook's headquarters each received a speeding ticket while driving home from work,

> they'd each owe the government the same amount of money. Mr. Zuckerberg wouldn't bat an eye.
>
> The janitor is another story.
>
> For people living on the economic margins, even minor offenses can impose crushing financial obligations, trapping them in a cycle of debt and incarceration for nonpayment. In Ferguson, Mo., for example, a single $151 parking violation sent a black woman struggling with homelessness into a seven-year odyssey of court appearances, arrest warrants and jail time connected to her inability to pay. [R34]
>
> Read the article and the reader's comments. If you were to post a comment at the website what would you say? You might consider comparing the fine structure to the graduated income tax.

Exercise 7.8.53. [U][N] Capturing fog.

On June 7, 2018 *The Boston Globe* reported on an MIT project to capture the water in fog.

> Varanasi said a typical 600-megawatt power plant consumes as much water annually as a city of 100,000 people, losing 750 million gallons to the air, 20 to 30 percent of which would be fog droplets.
>
> Varanasi envisions his system capturing 150 million gallons from the hypothetical plant. Varanasi didn't go into the details of what the system installation and operation would cost but said it would need just $10,000 in electricity to run. [R35]

(a) Estimate or search to find out how much water 100,000 people use annually.

At their website `water.usgs.gov/edu/qa-home-percapita.html` the USGS says that "Estimates vary, but each person uses about 80-100 gallons of water per day."

Calculating with 350 days per year 100,000 people use between 2.8 and 3.5 billion gallons per year.

(b) What fraction of the water consumed by the power plant is lost?

750 million is one fourth of 3.5 billion, so about 25 percent of the water is lost.

I wonder where the other $3/4$ goes.

(c) What fraction of the lost water would this system capture?

150 million is one fifth of 750 million.

(d) What is the cost in electricity to capture one gallon of the water?

$$\frac{\$10,000}{150 \times 10^6 \text{ gallons}} = 0.00007 \frac{\$}{\text{gallon}}$$

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