11

Probability

Pierre de Fermat and Blaise Pascal invented the mathematics of probability to answer gambling questions posed by a French nobleman in the seventeenth century. We follow history by starting this chapter with simple examples involving cards and dice. Then we discuss raffles and lotteries, fair payoffs and the house advantage, insurance and risks where quantitative reasoning doesn't help at all.

Chapter goals:

Goal 11.1. Compute probabilities for games of chance by counting outcomes.

Goal 11.2. Calculate fair price of a bet as a weighted average.

Goal 11.3. Calculate house advantage as (payout)/(income).

Goal 11.4. Understand insurance as a lottery.

11.1 Equally likely

In its everyday qualitative meaning "probably" is just a synonym for "likely" or "I think so but I'm not sure." In this chapter we start with simple examples where we can make "probably" quantitative by counting the possibilities.

To think about the chance of some particular event involving coins, dice, cards or raffles happening, count the possible equally likely outcomes, then count how many match what you're looking for and write down the appropriate fraction.

- The probability of heads when tossing a fair coin is $\frac{1}{2}$.
- The probability of rolling a 6 with a fair die is $\frac{1}{6}$.
- The probability of drawing an ace from a well shuffled deck is $\frac{4}{52}$.

In words:

$$\text{probability of an event} = \frac{\text{number of outcomes that match the event}}{\text{number of possible outcomes}}.$$

Writing probabilities as fractions helps you remember what they mean. But since they're just numbers, we can write them as decimals if we wish. Since they are numbers between 0 and 1, we often express them as percentages.

- The probability of heads tossing a fair coin is $\frac{1}{2} = 0.5 = 50\%$.
- The probability of rolling a 6 with a fair die is $\frac{1}{6} \approx 0.167 \approx 17\%$.
- The probability of drawing an ace from a well shuffled deck is $\frac{4}{52} \approx 0.077 = 7.7\%$.

Events that can never happen have probability 0. Events with probability 1 are certain to happen.

- The probability of rolling a 7 with a die is $\frac{0}{6} = 0 = 0\%$. It doesn't matter whether the die is fair or not.
- The probability of drawing a heart, a club, a spade or a diamond from a deck of cards is $\frac{52}{52} = 1 = 100\%$. It doesn't matter whether the deck is well shuffled or arranged in some nice order.

There are other probability problems you can solve by counting, as long as you're careful to count the right things.

Many state lotteries offer a prize if you pick the right six numbers in the some range. The numbers must be different, with no repetitions, but the order in which you pick them doesn't matter. To find the probability that your pick will win you have to count how many ways there are to pick six numbers. That's a problem for a math course more advanced than this one: the answer is 20,358,520 when the range is numbers from 1 to 52. So the probability of winning pick-six is about one twenty-millionth. If twenty million people play, expect about one winner.

We will have much more to say about lotteries in Section **??**.

11.2 Odds

Another way to describe a coin toss is to say "the odds are fifty-fifty." Heads and tails are equally likely — the odds are even.

Here are the odds for some gambling events; we usually write odds with a colon (:) and read the colon out loud as "to".

- The odds for rolling a 6 with a fair die are 1 : 5 or one to five. The odds against are 5 : 1, or five to one.
- The odds for drawing an ace from a well shuffled deck are 4 : 48, or 1 : 12. The odds against are twelve to one.
- The odds for heads tossing a fair coin are 1 : 1.

These examples illustrate how to find the odds for an event when you can count the equally likely possibilities and decide which ones are favorable. You compute

$$\text{(number of favorable cases)} : \text{(number of unfavorable cases)} .$$

The odds against the event are

$$\text{(number of unfavorable cases)} : \text{(number of favorable cases)}.$$

Odds are fractions in disguise, so the odds against drawing a spade from a deck of cards may be expressed as 39 : 13 (counting all the possibilities) or simply as 3 : 1 (three to one).

The odds against a winning pick-six ticket are about 20 million to 1.

You can convert back and forth between odds and probabilities. Since the odds against drawing a spade are 39 : 13, the probability that you won't draw a spade is $\frac{39}{52} = \frac{3}{4}$. In general, if the odds for an event are $a : b$ then its probability is $a/(a+b)$.

If you start out knowing that you will draw a spade with probability 25% you know too that the probability that you'll draw a heart, a diamond or a club is 75%. With both those probabilities it's easy to find the odds: they are 0.25 : 0.75 for drawing a spade. That's just our old friend 1 : 3 in disguise. Gamblers usually describe bets in terms of odds rather than probabilities. We will use odds that way in Section **??**.

In general, if the probability of an event is p then the odds for that event are $p : (1-p)$. The odds against are $(1-p) : p$.

The few formulas in this section are just common sense. If you understand them you won't have to memorize them. If you try to memorize them without understanding them you may end up using them in the wrong places.

11.3 Raffles

Simple raffles are gambles with computable probabilities. Tickets are sold, some are chosen at random and the people who hold those tickets get prizes. You may be familiar with fundraising raffles run by school parent teacher organizations.

Suppose the PTO sells 500 tickets for a raffle with a single prize.

Since each of the 500 tickets has an equal chance of being selected, the odds of a ticket winning are 1 : 499, or 499 : 1 against. The probability that any particular ticket wins is $\frac{1}{500} = 0.002 = 0.2\%$, or two tenths of a percent.

The probability that a particular person wins may be different. If you buy 10 tickets then you win with probability $\frac{10}{500} = 0.02 = 2\%$. If you don't play, the probability is 0. If you buy all of the tickets then you win with probability $\frac{500}{500} = 1 = 100\%$.

Now let's connect probability with money, as the inventors of the mathematics of probability did centuries ago. Suppose the PTO wants to offer a \$1000 prize to the winner. Then the *fair price* of a ticket is what it would cost if all the money collected were distributed as prizes:

$$\text{fair price} = \frac{\text{total prize money}}{\text{number of tickets}} = \frac{\$1,000}{500 \text{ tickets}} = 2\frac{\$}{\text{ticket}}. \tag{11.1}$$

Using what we learned in Chapter **??** we can rewrite this computation as a weighted average. One of the tickets is worth \$1000; the others are worthless, so

$$\begin{aligned}
\text{fair price} &= \frac{\text{total value of tickets}}{\text{number of tickets}} \\
&= \frac{499 \times \$0 + 1 \times \$1000}{500} \\
&= \frac{499}{500} \times \$0 + \frac{1}{500} \times \$1000 \\
&= \text{probability of losing} \times \text{value of losing ticket} \\
&\quad + \text{probability of winning} \times \text{value of winning ticket} \\
&= 0.998 \times \$0 + 0.002 \times \$1000 \\
&= \$2.
\end{aligned}$$

In the fourth line of the computation the ticket counts disappear. The fair price is the weighted average value of a ticket, weighted by the probabilities for each kind of ticket.

That average is the fair price of a ticket because all the money collected is returned in prizes. That may make for an exciting evening at the PTO meeting, but it won't raise any money. So the PTO decides to charge \$3.00 for each ticket, keep the prize at \$1,000, and use the other \$500 to buy classroom supplies for the kids.

Since the total prize money and the number of tickets have not changed, the fair price is still \$2. So on average each ticket loses

$$\text{cost of ticket} - \text{fair price of ticket} = \$3 - \$2 = \$1.$$

Of course you never lose exactly one dollar with one ticket. You either collect \$1000 for a net gain of \$997 or get nothing and lose your \$3 bet.

Yet another way to calculate the average loss is to see that the prize is just 2/3 of what the PTO collects, so the fair price is 2/3 of the \$3 cost, or \$2. Then on average each ticket loses the other 1/3, or \$1.

Would you buy a \$3 ticket when the fair price is just \$2, knowing that on average you will lose \$1.00? Perhaps. Even though you're very likely to lose your three dollars you can feel good about supporting the school. Maybe the thrill you get anticipating what you will do with the prize if you win despite long odds makes the probable loss more bearable.

11.4 State lotteries

On July 10, 2016 Jeff Jacoby wrote a column in *The Boston Globe* about the possibility that the Massachusetts state lottery might begin online ticket sales.

> Massachusetts officials boast that the state lottery is the nation's most successful. Some \$5 billion in lottery tickets were sold last year, a record high. On a per capita basis, lottery sales in Massachusetts — the amount spent on scratch tickets, the Numbers Game, Megabucks, and all the rest — averaged \$740. That is a stunning number. The 43 states and District of Columbia that operate lotteries did about \$70 billion worth of business last year, which averages out to \$230 in gambling revenue for every man, woman, and child within their borders. Thus sales in Massachusetts were more than three times the US average.
>
> ...
>
> State lotteries are often justified as an effective, yet voluntary, means of raising money to pay for public services. In Massachusetts last year, \$945 million in State Lottery profit was disbursed as local aid to the Commonwealth's cities and towns. [R**??**]

As usual, we start thinking about these numbers with a quick sanity check. Do the per capita amounts match the totals?

Estimating the U.S. population in 2016,

$$320 \text{ million people} \times 230 \frac{\$}{\text{person}} = 70.4 \text{ billion } \$$$

which is close enough to the \$70 billion in the article.

For Massachusetts,

$$6.5 \text{ million people} \times 740 \frac{\$}{\text{person}} = 4.8 \text{ billion } \$$$

which is close enough to the \$5 billion in the article.

	Income	Apportionment of funds		
	Ticket sales (excluding commissions)	Prizes	Administration	Proceeds available
Massachusetts	5,005,635	3,641,351	100,590	1,263,694
U.S.	66,885,544	42,893,054	3,125,938	20,900,504

Table 11.1. Massachusetts and U.S. state lotteries (2015) [R**??**]

The payoff rules for the various games are very complex, and vary widely from game to game. Fortunately, the Census Bureau provides the total amount returned in prizes so we can calculate the average fair price of a dollar ticket. Table **??** shows $66.9 billion for total U.S. state lottery revenues and $1.3 million for the net proceeds available in Massachusetts. These don't quite match Jacoby's figures, but they are in the same ballpark.

Massachusetts players spent $5 billion on tickets and received $3.6 billion in prizes. The average return was thus

$$\frac{3.6 \text{ billion prize dollars}}{5 \text{ billion purchase dollars}} = 0.72 \frac{\text{prize dollars}}{\text{purchase dollar}}.$$

The fair price for a one dollar Massachusetts ticket is just about 72 cents.

With that figure we can estimate the probability of winning when we know the prize structure. For example, for a single million dollar payout the Lottery Commision will have to sell

$$\frac{\$1,000,000}{0.72 \text{ \$/ticket}} \approx 1,400,000 \text{ dollar tickets}$$

in order to pay out 72% in winnings. Therefore the odds that a ticket wins are 1:1,400,000. You would have to buy 700,000 one dollar tickets for a 50% chance at the million dollar prize. To add insult to injury, if you won you would have to pay federal income tax on your winnings.

Using the government's data, the nationwide average fair price for a dollar ticket was

$$\frac{42.9 \text{ billion prize dollars}}{66.9 \text{ billion purchase dollars}} = 0.64 \frac{\text{prize dollars}}{\text{purchase dollar}},$$

so the Massachusetts gamblers are a little better off, per dollar. But they spend many more dollars. Figure **??** breaks down the total $66.9 billion by state.

States use lottery revenue to help balance their budgets, so you can think of lotteries as a kind of tax. What kind? Income and property taxes are roughly proportional to income or property values. Sales taxes are roughly proportional to sales. But prosperous citizens don't play the lottery in proportion to their prosperity, so a lottery is in effect a regressive tax . We close this section with a further quote from Jacoby:

> But it is odious for government to raise money by preying on the poor and the foolish. Everyone knows that lottery tickets are most frequently purchased by people least able to afford them.

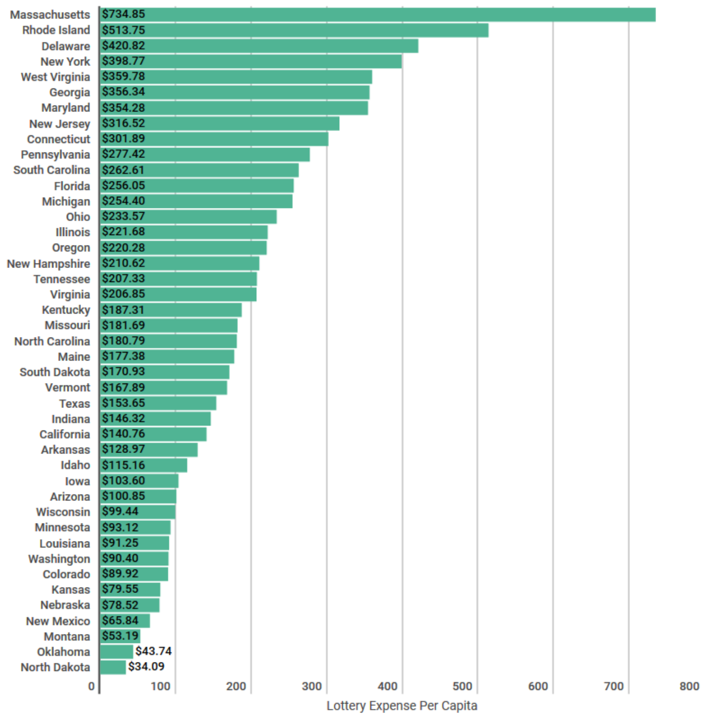


Figure 11.2. State per capita lottery expenses (2016) [R??]

11.5 The house advantage

Raffles and lotteries are designed to make money. So is casino gambling — for the casinos. They make a profit, and states tax the proceeds to raise revenue.

Before you lay down your bet at a casino, you should think about how much you will pay to play — the difference between a dollar bet and fair price of that bet (the average amount returned to you for your dollar). That difference is called the *house advantage*.

In Section **??** we discovered that the house advantage on state lotteries averages about 37%. At gambling casinos it's much smaller. As in the state lotteries, the house advantage varies from game to game. It's the highest (about 10%) for slot machines — and there is no way to know that when you decide to play. But for roulette we can actually calculate the house advantage.

A fair roulette wheel is a circle divided into 36 equal wedges numbered from 1 to 36, colored alternately red and black. A ball runs around the rim of the wheel, slowing down until it falls into a random wedge. Before the wheel spins you place your bet, perhaps:

- on the number 17 ("straight-up"), with a winning probability of $\frac{1}{36}$. The odds are 35 to 1 against.
- on red, with a winning probability of $\frac{18}{36} = \frac{1}{2}$. Even odds.

- on odd, at even odds.
- on one of the numbers 1 through 12 (a "dozen bet"), with a winning probability of $\frac{12}{36} = \frac{1}{3}$. Two to one against.

What would be a fair return on a $1 bet?

- If you bet straight-up the payoff should be $36.
- If you bet on red, the payoff should be $2.
- If you bet on odd, the payoff should be $2.
- For a dozen bet the payoff should be $3.

There are several ways to see that these are fair. We'll work them out with the $36 payoff for the straight-up dollar bet on a single number.

- Imagine the spin of the wheel as a raffle with 36 tickets. A dollar bet on 17 is like buying one of the tickets. Imagine that others have bought the other 35 for $1 each. Then the casino has collected $36. The fair thing to do would be to pay that to the winner — then all the money collected is awarded as prizes.
- Using the technique we learned in Section **??**, we can check the numbers in this equation:

  winning probability × winning payoff + losing probability × losing payoff.

  For the straight up bet that equation says
  $$\frac{1}{36} \times \$36 + \frac{35}{36} \times \$0 = \$1$$
  which is a perfectly fair average return on a $1 bet!
- What happens when you play for a long time? Since you pay $1 for each spin of the wheel and win about $\frac{1}{36}$ of the time you should collect $36 for each win in order to break even in the long run.
- The odds for winning are 1 : 35. Your $1 bet on 17 is a bet against the casino. They put up $35 to match your $1. The winner takes all $36.

The other computations (to check the fair payoffs for bets on red, or odd, or 1-12) work the same way.

In real life the casino must cover expenses, pay the state its share of the take and still turn a profit, so the average value of a bet must be less than the fair price. The difference is the house advantage.

In the PTO raffle the organization assumes it sells all the tickets and decides how much of what it collects to return as prizes. But the casino can't count on people betting on all the numbers, and can't know how many people will bet.

Figure **??** shows how they collect the house advantage in roulette. The picture shows a wheel with an extra green wedge numbered 0. An American roulette wheel will have another green wedge numbered 00.

Figure 11.3. The house advantage in roulette [R**??**]

The casino uses the old fair price payoffs: \$36 for a winning \$1 straight-up bet on 17. But the extra wedges change the probabilities. Here is the calculation for an American wheel with 38 wedges:

$$\frac{1}{38} \times \$36 + \frac{37}{38} \times \$0 = \$0.94736842105 \approx \$0.95$$

which means that on average you lose a little more than a nickel of every dollar you bet. The house advantage is just over 5.25%.

Does this mean you shouldn't play? Not necessarily. You may be willing to pay the house advantage in return for the thrill of the gamble. But before you do, you should understand the odds for the game you choose.

There are casino games in which a skilled player can win — slowly, and with great effort. At the poker table you are competing with other gamblers, not with the house, which pays its expenses and profits by taking a fraction of the ante or pot on each deal. So the house always wins, but a skilled poker player can win too by beating the other players.

In principle, you can also win at blackjack. We'll think about why in the next chapter.

11.6 One-time events

In our discussions so far we've assumed each example is "fair" (even if payoffs weren't) — coins and dice and roulette wheels are properly balanced, decks of cards are properly shuffled, no one peeks when drawing the winning raffle ticket. In each case all possible outcomes are equally likely so we could compute probabilities just by counting cases.

To test for whether a particular coin or die is really "fair" you could imagine repeating an experiment many times. A fair coin should come up heads about half the time (but not exactly half the time, which would be very unlikely). A fair die should show a 5 about 1/6 of the time. We'll return to this topic in Section **??**.

Horse	Bets (K$)
Barbaro	239.0
Spend a Buck	333.2
Donerail	18.6
Twenty Grand	904.4
Apollo	155.0
Dark Star	66.3
total	1,716.5

Table 11.4. Race of champions

There are many situations in real life where probabilities and odds appear but can't be computed by simple counting or checked by repeated experiments. Will the Chicago Cubs win the World Series? Which horse will win the Kentucky Derby? Who will be elected? Will it rain tomorrow?

Suppose you bet your Chicago friend that the odds against the Cubs winning the World Series are 99 : 1. You put up $99, she puts up $1 and the winner takes home $100 when the season is over. That means that (in principle) you believe that probability of that Cubs World Series win is just $1/100$. (Those might have been the right odds before 2017, when the Cubs won the World Series for the first time in more than a century.)

When lots of people have an opinion they are willing to bet on, they can decide the probability collectively.

There's a way in which many state lottery payoffs depend on what the bettors think: the total prize money for a winning pick-six combination is divided among the people who bet on that combination. The odds for any particular number combination don't change, but the payoff does. Exercise **??** pursues this idea.

In horse racing the odds at the track depend on the bets placed, in what's called *parimutuel* betting. Most readers of this book won't be playing the horses, and those who do will (or should) know all about this kind of betting. We discuss it here anyway since it provides an example where we can actually see how the bets determine the odds.

Before the race the punters place their bets at the tote. ("Punter" and "tote" are racing terms. You can look them up if you don't know what they mean.) After the race the track skims its *take* or *commission* — a percentage of the total amount bet. The winning bettors share the remainder in proportion to the amount each bet.

Table **??** shows the amount bet on each of six horses in an imaginary race. The horses are real — all winners of the Kentucky Derby — but we made up the numbers.

The favorite horse is Twenty Grand precisely because more people have bet on him to win.

Since the total amount bet is $1,716,500, the collective wisdom at the track says that Apollo will win with probability $155,000/1,716,500 = 0.0903000291 \approx 9\%$. The fair payoff is $1,716,500/155,000 = 11.0741935 \approx 11$ dollars per dollar bet.

That corresponds to odds against of about 10 to 1. If Apollo wins, each dollar bet will collect $11: the original dollar plus the ten the other bettors put up in vain.

We had fun choosing the amounts bet on each of these six horses so that the odds of each are close to their odds in the Derby they won. We've included the longest shot of all, Donerail, and a favorite, Twenty Grand, who ran at less than even odds.

The 10 : 1 odds for Apollo do not take into account the race track's commission. We don't know how much that was, or even whether it was the same in all six races. Suppose that in this fantasy race it's 10%.

Suppose Apollo wins. The track pays 90% of the take to those who bet on Apollo — $0.9 \times \$1,716,500/\$155,000 = \$9.96677\ldots \approx 10$ dollars per dollar, instead of 11 dollars per dollar. The odds are effectively just 9 : 1 against. The track makes money by lowering the odds, which no longer reflect the probabilities determined by the bets.

The moral of the story: you can win at the race track if you really know better than most people which horse is likely to win. There's that word "likely" again - you need to know a lot about the horses and play a lot for your knowledge to pay off. Perhaps the best way to win is to sell suckers a system ....

11.7 Insurance

When you buy insurance you're gambling. In this case the gamble is one you hope to lose — you don't want to get sick, or have your house burn down, or total your car. In each of those situations you've made a small advance payment you hope and expect to lose in order to cover your losses when a catastrophic event with small probability happens.

Insurance companies estimate probabilities in order to determine the fair price for their policies, then add what they need to cover their administrative expenses and make a profit - their "house advantage." In the long run, on average, their customers never get all their money back. Therefore you want to think things through when you're deciding whether to buy insurance for more than the fair price. Sometimes you may be better off accepting the risk yourself.

Here's a sample of the kind of advice you can find on the web. It's from Liz Pulliam Weston, writing for *MSN Money*.

> Say you have a 10-year-old Honda that's worth $4,000 in a private-party sale and have a $500 deductible. Your risk is $3,500. If your premiums for collision and comprehensive are more than $350 a year, it may be wiser to bank that money toward a newer car. [R**??**]

If we make a simple assumption we can think about this using probabilities. Suppose that the only kind of accident to worry about is one that totals the car. Then Weston's advice is reasonable if you think that the probability that you'll have such an accident is less than 10%. Here's why. Imagine that the insurance policy is a lottery ticket, which "wins" if you have an accident. A winning ticket is worth $3,500. If you think you have a 10% chance of winning, then the fair price (for you) is $350. If you think your chance of totaling your car is less than 10% then the fair price is more than $350, so perhaps you shouldn't buy the insurance.

Of course the real decision isn't this easy. You should take into account the fact that your accident might not total the car. You have to think about making this decision every year — sometimes your car will be worth more than $4,000, sometimes less. But the principle is clear. If the premiums are very high compared to your estimate of your risk, you should consider not buying collision and comprehensive insurance. Over the course of a driving lifetime you will probably save money.

However, there are often good reasons to pay more than the fair price for insurance. If you don't have the money to replace a totaled car and you must have one, then you need that insurance. Even if you have the money, the cost to you of a large loss may be more than you can afford, or may feel like more than the dollar amount.

For a discussion of answers to the question "Why buy insurance?" visit `money.stackexchange.com/questions/54561/why-buy-insurance`.

Sometimes you may be required to buy insurance. In order to drive, you must carry liability insurance to cover the cost of injuries to others in an accident you caused. If you have a mortgage on a house the bank will insist on fire insurance to protect their interest in the money they've lent you. The taxes you pay to support the police and fire departments can be considered a kind of insurance. You will probably never need their services, but you want them to be there when you do. Healthy people buy health insurance (and may even be required to do so) to spread the cost of catastrophic medical bills.

George Bernard Shaw wrote about this in "The Vice of Gambling and the Virtue of Insurance" [R**??**] . There's a section on health insurance that's the clearest argument we've seen for single payer "medicare for all". Too bad it was written a century ago by a socialist.

11.8 Sometimes the numbers don't help at all

About forty years ago Joan Bolker had to decide whether to invest three years of hard work in hopes of earning a clinical psychology license.

Only after more than two thousand hours of clinical internships (which she would have to arrange) could she petition to have her doctorate in education count as appropriate postgraduate preparation for her new career. Only if that petition were granted would she be allowed to take the psychology licensing examination, much of which covered material she had not studied in any course.

Clearly the odds were long. She faced a significant investment of time, energy and lost income, with an unknown and hard to estimate probability of success at the end. She took the risk. She won her gamble, with a combination of talent, persistence and luck.

The moral of the story: sometimes numbers don't help. "Not everything that can be counted counts, and not everything that counts can be counted." (A quote often (wrongly) attributed to Albert Einstein). [R**??**]

In this case there was no way to quantify the costs, the benefits and the probabilities in order to make what might look like a rational choice. The kind of back-of-an-envelope probability calculations we've studied about playing the lottery or buying insurance are often of little help when making life-changing one-of-a-kind choices.

11.9 Exercises

Exercise 11.9.1. [S][R][Section **??**][Goal **??**] What's in a name?

(a) What is the probability that the name of a state (of the United States) chosen at random begins with the letter "A"?

(b) What is the probability that the name of a state (of the United States) chosen at random begins with the letter "Z"?

(c) How much more likely is it that a state name begins with "M" than with "A"?

(a) What is the probability that the name of a state (of the United States) chosen at random begins with the letter "A"?

Four state names begin with "A" so the probability is 4/50 or 8%.

(b) What is the probability that the name of a state (of the United States) chosen at random begins with the letter "Z"?

No state names begin with "Z" so the probability is 0.

(c) How much more likely is it that a state name begins with "M" than with "A"?

Since eight state names begin with "M" and four with "A", it's twice as likely: 16% instead of 8%.

Exercise 11.9.2. [U][Section **??**][Goal **??**] "Probably" in everyday English.

(a) Use the index to this book to find places where we used the words "probably" or "likely" other than in the chapters devoted to studying probability. Discuss the meaning of the word there. When it makes sense, provide a numerical estimate of the probability.

(b) Do the same for two or three occurrences of "probably" or "likely" in the media.

Exercise 11.9.3. [S][Section **??**][Goal **??**] Is it safe to swim?

In an article in *The Boston Globe* reprinted from the *Washington Post* on March 4, 2012 you could read a story headlined "Possible cut to beach testing a health threat, critics say". The story reports on Environmental Protection Agency estimates that say that the average person goes to a beach, lake or river about 10 days a year, and that about 3.5 million people get sick from splashing in bacterial contaminated water. [R**??**]

What is the probability that a visit to the beach will make you sick?

To estimate the total number of beach visits in a year I will multiply the U.S. population (about 300 million in 2012) by the average number of visits per person (10, which seems large to me). That gives 3 billion visits.

Of those visits, 3.5 million result in illness. That's 0.001166, or about one tenth of one percent. So the probability is 0.001. It's not reasonable to use more precision than that.

Exercise 11.9.4. [S][Section **??**][Goal **??**] It's a horse race.

Use the data in Table **??** to compute

(a) The odds and payoff for Donerail, the long shot.

(b) The odds and payoff for Twenty Grand, the favorite.

(c) The payoff for these two horses if the track takes a 10% commission before paying off any bets.

[See the back of the book for a hint.] You might want to do this exercise in Excel. Then you can see the odds for all the horses, and see how the payoffs change when you change the track's take.

A student solution.

(a) The odds and payoff for Donerail, the long shot.

$18,600/1,716,500 = 0.01083$, which is about 1% probability. The payoff is $\$1,716,500/\$18,600 = \$92.28$ per dollar bet. So the odds are about 91 to 1.

(b) The odds and payoff for Twenty Grand, the favorite.

$\$904,400/\$1,716,500 = 0.52688$ which is a 53% probability. The payoff is $\$1,716,500/\$904,400 = \$1.90$ per dollar bet. The odds are less than 1 to 1.

(c) The payoff for these two horses if the track takes a 10% commission before paying off any bets.

A 10% take leaves a remaining purse of $\$1,716,500 - \$171,650 = \$1,544,850$. The payoff is $\$1,544,850/\$18,600 = \$83.06$ per dollar bet on Donerail.

$\$1,544,850/\$904,400 = \$1.71$ per dollar bet on Twenty Grand

I guess recalculating the probabilities actually doesn't need to happen here, or maybe doesn't make any sense.

Exercise 11.9.5. [S][Section **??**][Goal **??**] Extended warranties.

The list below from `tv.about.com/od/warranties/a/buyexwarranty.htm` outlines a set of points to think about when deciding whether to buy an extended warranty for your new TV. We think something important is missing from this list. What is it?

(1) Value of item being purchased
(2) Price of extended warranty
(3) Length of manufacturer's warranty
(4) Length of extended warranty and date coverage begins [R**??**]

What's missing is information on how reliable the TV is. In order to make a computation comparing the cost of the warranty with the cost of repair or replacement I need an estimate of the probability that the TV will break after the regular warranty expires.

Exercise 11.9.6. [S][R][Section **??**][Goal **??**] Which average?

In the raffle discussed in Section **??** there are 500 tickets and a $1000 prize. We found that the average value of a ticket was $2.

(a) Which average is that — mean, median or mode?

(b) Compute the other two "average" ticket values.

(a) The $2 "average value" is the mean.

(b) Since 499 of the tickets are worthless the most common ticket value is $0, so that's the mode. The median ticket value is also $0 since half the tickets are worth that or less, half that or more.

Exercise 11.9.7. [S][R][Section **??**][Goal **??**] Multiple prizes.

Suppose a lottery with 1,000,000 tickets has a first prize of $200,000, three second prizes of $60,000 each and 100 third prizes of $200 each.

(a) What is the probability that a ticket wins the first prize?

(b) What is the probability that a ticket wins some prize?

(c) What is the fair price of a ticket?

(d) How much should the state charge for a ticket if it needs 10% of the revenue for overhead and wants to make $500,000 profit?

(a) What is the probability that a ticket wins the first prize?
One in a million, or 0.000001.

(b) What is the probability that a ticket wins some prize?
104/1,000,000.

(c) What is the fair price of a ticket?

$$\text{fair price} = \frac{\text{total prizes}}{\text{tickets sold}} = \frac{\$200,000 + 3 \times \$60,000 + 100 \times \$200}{1,000,000} = \$0.40.$$

(d) How much should the state charge for a ticket if it needs 10% of the revenue for overhead and wants to make $500,000 profit?

Well, 90% of what they collect must cover $500,000 in profit plus $400,000 in prizes. So 90% of what they collect is $900,000. That means they need to collect a million dollars. Since they are selling a million tickets. each ticket should cost a dollar.

Exercise 11.9.8. 1996 was a long time ago.
Moved to `www.commonsensemathematics.net/csmexercises.pdf`

Exercise 11.9.9. [U][Section **??**][Goal **??**] 1996 was a long time ago.

> Lotteries rank first among the various forms of gambling in terms of gross revenues: total lottery sales in 1996 totaled $42.9 billion. 1982 gross revenues were $4 billion, representing an increase of 950% over the preceding 15 years, 1982-1996.
>
> Lotteries have the highest profit rates in gambling in the U.S.: in 1996, net revenues (sales minus payouts, but not including costs) totaled $16.2 billion, or almost 38% of sales. They are also the largest source of government revenue from gambling, in 1996 netting $13.8 billion, or 32% of money wagered, for governments at all levels. [R**??**]

The quotation that starts the section on lotteries in the first edition of *Common Sense Mathematics* comes to us courtesy of the University of North Texas CyberCemetery:

> The University of North Texas Libraries and the U.S. Government Printing Office, as part of the Federal Depository Library Program, created a partnership to provide permanent public access to the Web sites and publications of defunct U.S. government agencies and commissions. This collection was named the "CyberCemetery" by early users of the site. [R**??**]

The bookkeeping[1] for analyzing these numbers is

$$\text{total from ticket sales} = \text{ prizes awarded } + \text{ overhead } + \text{ net revenue to state.}$$

In 1996 gross revenues — that is, ticket sales, dollars bet — were $42.9 billion.

The $16.2 billion in the second paragraph is "sales − payouts", so the payouts must be $\$42.9 - \$16.2 = \$26.7$ billion. Then

$$\frac{\text{payouts}}{\text{sales}} = \frac{\$26.7 \text{ billion}}{\$42.9 \text{ billion}} = 0.622377622 \approx 62\%$$

[1] One of our favorite words. We don't know another with three double letters in a row.

so for each lottery dollar in 1996, players got back (on average) a little more than 62 cents in prize money. That is the fair price of a one dollar ticket. The other 38 cents is the 38% of sales that count as total revenue for the government — the $16.2 billion not returned to bettors as prizes. Some of that money was overhead. After subtracting that, the net revenue available for other use was $13.2 billion.

Update the numbers from that quote (go back to Section **??**) so that you can rewrite the paragraph referring to a much more recent year than 1996.

Exercise 11.9.10. [S][W][Section **??**] [Goal **??**] Massachusetts Lottery statistics.

- The Massachusetts Lottery Commission reported that in 2006 they distributed over $761 million in Direct Local Aid to the Cities and Towns of the Commonwealth.
- From `www.masslottery.com/winners/faqs.html`

> What happens to the revenue which the Lottery generates from sales?
> 1. A minimum of 45% of revenues stays in the State Lottery Fund to be paid out in prizes. The Lottery's current prize percentage is over 69
> 2. A portion of revenues is transferred to the commonwealth's General Fund for the expenses incurred in administering and operating the Lottery. The administrative and operating expenses of the Lottery are appropriated by the legislature as part of the annual state budget. Operating expenses cannot exceed 15%. Currently, operating expenses are under 8%. These operating expenses include 5.8% in commissions and bonuses paid to the sales agents who sell the tickets and under 2% in administrative expenses due to Lottery operation.
> 3. After prizes and expenses, the remaining Lottery revenues (approximately 23%) is transferred to the Local Aid Fund and returned to the cities and towns of the Commonwealth in the form of local aid. [R**??**]

- Several years later, on January 5, 2011 *The Boston Globe* reported that about $26 million in tickets had been sold in hopes of winning the $355 million Mega Millions jackpot, and that "the tickets have raised $11 million for cities and towns". [R**??**]

(a) Sketch a pie chart showing how the money collected by the Lottery Commission was distributed among prizes, overhead and aid to Cities and Towns. Label each of the three slices with its percentage, and one of the slices with an amount of money.

(b) What was the total dollar amount collected by the Lottery Commission in 2006?

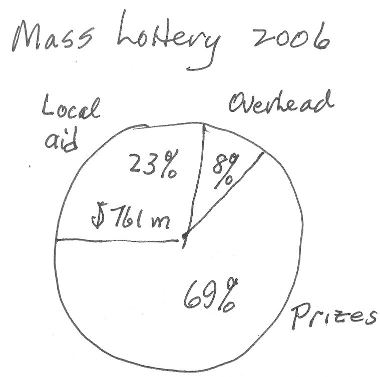
(c) What was the fair price of a $5 ticket?

(d) How much on average did people in Massachusetts spend on lottery tickets in 2006? On average, how much did they get back in prizes? Is this "average" the mean, the median or the mode?

(e) Does the 2011 payout for the 16-draw series match the prize percentage reported in 2006?

(a) Sketch a pie chart showing how the money collected by the Lottery Commission was distributed among the three categories prizes, overhead and aid to Cities and Towns. Label each slice with its percentage, and one of the slices with an amount of money.

Here is my sketch. The one actual amount available is the $761 million in local aid.



(b) What was the fair price of a $5 ticket?

Assuming as we did before that they distribute 69% of what they collect, the payoff (on the average) from a $5 ticket would be $\$5 \times 0.69 = \$3.45$. So $3.45 is the fair price. The actual price of $5 is higher because the Commonwealth takes part of the proceeds for itself before distributing the prizes.

(c) What was the total dollar amount collected by the Lottery Commission in 2006?

The Lottery Commission distributed $761 million in direct local aid. They say that's 23% of what they collected. That means

$$0.23 \times \text{ (total collected) } = \$761 \text{ million}$$

so the total collected was

$$\frac{\$761 \text{ million}}{0.23} = \$3308.69565 \text{ million} \approx \$3.3 \text{ billion}.$$

Sanity check: since 23% is about one fourth, the answer is about four times $761 million, as it should be. In my chart I drew the local aid slice as about a quarter of the pie.

(d) How much on average did people in Massachusetts spend on lottery tickets in 2006? On average, how much did they get back in prizes? Is this "average" the mean, the median or the mode?

The population of Massachusetts in 2006 was about 6.5 million. (`lmi2.detma.org/lmi/pdf/PopbyAgeandSexMA.pdf`) so the average citizen of the Commonwealth spent

$$\frac{\$3.3 \text{ billion}}{6.5 \text{ million people}} = 507.692307692 \frac{\$}{\text{person}} \approx 500 \frac{\$}{\text{person}}$$

on lottery tickets. I find that a scary fact.

On average, they got the fair price back in prizes: $345 per person. But this average (the mean) is misleading. The mode and the median are much smaller. A few people got a lot of money. Most got nothing, or nearly nothing since there are some small prizes to keep people playing.

(e) Does the 2011 payout for the 16-draw series match the prize percentage reported in 2006?

The 2011 prize percentage was

$$\frac{\$11 \text{ million}}{\$26 \text{ million}} = 0.42$$

or 42%. That's much less than the average 69% return in 2006.

Exercise 11.9.11. [S][Section **??**][Goal **??**] Megabucks changes the odds.

The first two paragraphs of an article in *The Boston Globe* on March 21, 2009 said that

> Like anyone who plays the lottery, Dean Thornblad was hoping to get rich quickly. He studied the odds of winning the various games before shelling out $150 for three season tickets that automatically enter him in twice-weekly drawings of Megabucks. At 1 in 5.2 million, the odds of hitting the jackpot, long by any standard, seemed to him at least "somewhat imaginable."
>
> But even his boundless optimism is being stretched by the lottery's latest proposal. The agency, under mounting pressure to return more money to cash-strapped cities and towns, is planning to make the odds of winning even slimmer, reducing them to 1 in 13.9 million beginning May 2, by making players match six numbers between 1 and 49, instead of six between 1 and 42. [R**??**]

What has happened to the expected value of Thornblad's ticket?

Thornblad's probability of winning has decreased from $1/(5.2 \times 10^6)$ to $1/(13.9 \times 10^6)$. The ratio of those two probabilities is $5.2/13.9 \approx 0.37$. That means the (very small) probability that he wins now is just 37% of what it used to be, so his ticket is now worth 37% of what it was worth before. I don't know its actual value since I don't know the actual jackpot. Maybe that will change if the number of people playing changes.

Exercise 11.9.12. [S][C][Section **??**][Goal **??**] Uncommon numbers.

In many state lotteries the customer picks the numbers she thinks will win. The prize is then divided among all the people who happened to pick the winning numbers. Much as we try to analyze only real situations, the real Massachusetts lottery is too complicated for this class. (Many people find it too complicated to choose the numbers they want to bet on, and elect "quick picks" instead.) So this question is about an imaginary lottery.

Here is how our lottery works. Tickets cost $1. Each person buying a ticket chooses the number between 1 and 100 that she thinks will win. When all the tickets have been sold, the state picks a number at random between 1 and 100. All the people who have chosen that number divide 70% of the total collected among themselves. (The other 30% the state uses for overhead and local aid.) So the fair price for a $1 ticket is $0.70 or 70 cents.

Of course the winners collect much more than the fair price (since the losers collect nothing). For example, if 1000 people bought tickets, 39 was the winning number, and 8 people chose 39, each would get $(\$1000 \times 0.7)/8 = \$87.50$.

If everyone buying tickets used "quick pick" then the 1000 tickets would (more or less) consist of 10 for each of the 100 numbers, ten people would have the winning number and the typical payoff would be $\$1000 \times 0.7/10 = \$70$.

Now that you've read this far and understood the game, we can ask an interesting question. Suppose you know that people are so afraid of the number 13 that no one ever picks it. You think (correctly) "If I buy a ticket and choose 13, I'm probably not going to win. But if I do win, I will win big because I won't have to share the prize." So every day you buy one of the 1000 tickets, and choose 13, knowing that no one else will. You lose with probability $99/100 = 0.99 = 99\%$ and win with probability $1/100 = 0.01 = 1\%$.

In the long run, how much money do you win (on the average) each day?

[See the back of the book for a hint.] You might find it easiest to answer this question by imagining that you played the lottery 100 days in a row.

At `blogs.wsj.com/numbersguy/lottery-math-101-801/` you can read more about this idea:

> Low numbers are particularly popular, some of them because birthdays are a popular source of numbers to play. Research conducted by Tom Holtgraves showed that bettors also avoid numbers with repeated digits, though these are just as likely to turn up in lotteries as numbers without. [R**??**]

For still more information, see "Q3.4: Can RANDOM.ORG help me win the lottery?" at `www.random.org/faq/#Q3.4`.

If I play the lottery 100 days in a row and I pick the number 13 each day I expect to win (about) once, since there are 100 equally likely numbers. When I win I am the only winner, so I get all 70% of \$1000, or \$700. Playing for 100 days costs me \$100, so I make \$600 every 100 days, or \$6 per day on average.

I can get the same answer with weighted averages, using the probabilities in the example for the weights: the average value of my one dollar daily ticket is

$$0.99 \times \$0 + 0.01 \times \$700 = \$7.$$

Subtracting the dollar I paid for the ticket nets me \$6 per day, on average.

Interesting note: The more people who play, the larger my average daily winnings (as long as no one else chooses 13).

Exercise 11.9.13. [R][Section **??**][Goal **??**] It's always 5.26%.

Compute the house advantage with an American wheel for each of the roulette bets in Section **??** to show that it's the same for each bet.

Exercise 11.9.14. [S][Section **??**][Goal **??**] Single zero roulette.

(a) Compute the house advantage for a roulette wheel with one extra wedge.

(b) Show that the house advantage in single zero roulette is approximately but not exactly half the house advantage in double zero roulette.

(a) Compute the house advantage for a roulette wheel with one extra wedge.

The house advantage in single zero roulette is

$$\frac{1}{37} = 0.02702702702 \approx 2.7\%.$$

(b) Show that the house advantage in single zero roulette is approximately but not exactly half the house advantage in double zero roulette.

The house advantage in double zero roulette is

$$\frac{2}{38} = 0.05263157894 \approx 5.25\%.$$

Half of that is a little less than 2.7%.

I could have done this without a calculator just by thinking about the fractions:

$$\frac{1}{2}\frac{2}{38} = \frac{1}{38} < \frac{1}{37}.$$

Exercise 11.9.15. [S][Section **??**][Goal **??**] Help this fellow out, please.

A questioner on the web has posted his roulette strategy. He says he will pick 19 numbers and bet on them. Since there are 38 spaces, he will win half the time, with a 35 to 1 payoff. So when he wins he's ahead $36 - 18 = 18$ dollars. Then he asks

> Am I missing something, or is it really that simple? [R**??**]

Answer his question.

Yes, you are missing something important. You bet a total of \$19. Half the time you will get \$36 back — the payoff at 35:1 on your winning \$1 bet. That's \$18 per spin, on average. In the long run you lose \$1 or $\$1/\$19 \approx 0.053 = 5.3\%$ of your money per spin. That's the house advantage; they'll be glad to take your bets.

Exercise 11.9.16. What you're counting counts.
Moved to `www.commonsensemathematics.net/csmexercises.pdf`

Exercise 11.9.17. [U] What you're counting counts.

(a) What is the probability that a random word in English begins with the letter "t"?

This is a question with several answers, which depend on how you select your "random word". You might count the words that begin with "t" in the dictionary. You might count those words in a newspaper, or on a website. There may be answers to the question on the web.

Estimate the answer in several ways. Do the various assumptions lead to approximately equal answers?

(b) "e" is the most commonly used letter in English. What is the probability that a random letter in an English text is "e"?

Attack this question as you did the previous one.

(c) What is "etaion shrdlu" and where does it come from?

Exercise 11.9.18. What is wrong with this estimate?
Moved to `www.commonsensemathematics.net/csmexercises.pdf`

Exercise 11.9.19. [S][Section **??**][Goal **??**] What is wrong with this estimate?

The following report appeared in the Offline column of *The New York Times* business section on March 8, 2008, where "Cubicle Coach" Marie Claire, says "take a chance" when considering whether to hire the ordinary candidate or one

> ...who has the potential to be great, but has an equal chance of being awful?
>
> "You have a 66.7 percent chance of a positive result," the coach writes. "Yes, the unknown could flop, but she could also a) do as well as the known, or b) actually be a star." [R**??**]

(a) What assumption is Claire making that leads her to her estimate of 66.7%?

(b) Suppose Claire is correct when she assumes that the probabilities of great and awful are equal. Show that the chance of a positive result (great, or just OK) is somewhere between 50% and 100%.

(a) What assumption is Claire making that leads her to her estimate of 66.7%?

There are three alternatives: safe bet, risky and a flop, risky and outstanding. Claire is assuming each is equally likely. In 2/3 if the cases you end up with someone who can do the job. Claire counts that as a "positive result".

(b) Suppose Claire is correct when she assumes that the probabilities of great and awful are equal. Show that the chance of a positive result (great, or just OK) is somewhere between 50% and 100%.

Assume "great" and "awful" are equally likely in the risky category and think about the probability of that category. If it's 1 (no ordinary candidates) then half the time you have a satisfactory outcome. If it's 0 (no risky candidates) then you're sure to have a satisfactory outcome.

Exercises added for the second edition.

Exercise 11.9.20. [S][R][Section **??**][Goal **??**][Goal **??**] What was fair in Texas?

The table at `www.ncsl.org/research/financial-services-and-commerce/lotteries-and-revenue-by-state-2010.aspx` lists the following data for the 2010 Texas lotteries:

Income	Apportionment of funds		
Ticket sales (excluding commissions)	Prizes	Administration	Proceeds available
\$3,542,210	\$2,300,182	\$184,980	\$1,057,048

What was the fair price of a \$1 ticket ?

Since

$$\frac{2,300,182}{3,542,210} = 0.649363533 \approx 65\%$$

Texas returned 65 cents on the dollar in lottery payouts in 2010. That would be the fair price of a dollar ticket.

Exercise 11.9.21. [U][Section **??**] Your state lottery.

Find and analyze the data for the most recent lottery in your state. Calculate the fair price of a dollar ticket and the per capita dollar sales figure.

Exercise 11.9.22. [S][Section **??**][Goal **??**] Differ by two

We found this question at `math.stackexchange.com/questions/1716651/roll-two-dice-what-is-the-probability-that-one-die-shows-exactly-two-more-than`

> Two fair six-sided dice are rolled. What is the probability that one die shows exactly two more than the other die (for example, rolling a 1 and 3, or rolling a 6 and a 4)? [R**??**]

Here's an answer by user `probablyme` (`math.stackexchange.com/users/290196/probablyme`):

To get yourself started, you could draw a table. The rows could be one roll, and the columns could be the other roll. Then the checkmark shows where the rolls are "two away" from each other.

	1	2	3	4	5	6
1			✓			
2				✓		
3	✓				✓	
4		✓				✓
5			✓			
6				✓		

Notice that, since all pairs are equally likely, we have a $8/36 = 2/9$ chance of being "two away". [R**??**]

Exercise 11.9.23. [U][S][Section **??**][Goal **??**] Stretching for the ball.

Peter Abraham reported on Jackie Bradley's catch in the Red Sox 2017 home opener:

> According to MLB's Statcast system, which tracks plays with high-resolution cameras and radar, Bradley had only a 55 percent chance of catching the ball and had to go nearly 30 yards to get there. [R**??**]

(a) How might the Statcast system come up with the "55 percent" estimate?

(b) Was this a lucky catch?

(a) How might the Statcast system come up with the "55 percent" estimate?

The system has data from video of many plays. They might have looked at all those in which the fielder had to run 30 yards and found the fraction where he managed to catch the ball.

(b) Was this a lucky catch?

I don't think so, for two reasons. First, to call it "lucky" I would want a success percent much smaller than 55%. Even then I would attribute success to skill rather than luck.

Exercise 11.9.24. [S][Section **??**][Section **??**] Spoiler alert?

On May 21, 2019 *The New York Times* reported on betting activity just before the final episode of *Game of Thrones*.

> Bran was no mere favorite: The final wagers for him to rule the kingdom were placed at 2-to-9 odds on one major offshore betting site, which implied that the probability he would prevail was almost 82 percent.

> That made for an expensive bet. For every \$45 risked, the profit would only be \$10. It was the sort of wager to make only by those who are pretty certain they are right. [R**??**]

Note that gambling odds like these are often quoted in the order opposite that we described in [Section **??**]. There the odds for the unlikely 6 in a die toss are 1 : 5. A gambler would say that as "5 : 1 against". So in this example, the odds of 2 : 9 mean that Bran's success is likely, not a long shot.

(a) Verify the quoted probability.

(b) If you bet \$100 on Bram what would you have collected?

(c) Why do you think the author of this article used a \$45 bet to illustrate his point?

(a) Verify the quoted probability.

Odds of 2:9 mean the probability of winning is $9/(2+9) = 0.818 \approx 82\%$.

(b) If you bet \$100 on Bram what would you have collected?

You would collect your bet along with \$2 for every \$9, so

$$\$100 + \frac{\$100}{\$9} \times \$2 = \$122.22.$$

(c) Why do you think the author of this article used a \$45 bet to illustrate his point?

Calculating the payoff is easiest for multiples of 9. 45 is the smallest multiple that makes the bet a nice round number.

Exercise 11.9.25. [S][Section **??**][Section **??**] Football.

> In May, 2019 *The New York Times* reported on the surprising success of Liverpool's soccer team in a match against Barcelona.
>
> Before the series started, Barcelona were the strong favorite to advance to the final, and the outcome of the first game validated that assessment. After that, someone who wanted to win \$100 betting on Barcelona needed to risk \$1,800 to do it. [R**??**]

What was the probability bookmakers assigned for the match after the first game?

An \$1800 bet returning \$100 in winnings would be fair if the probability of winning was

$$\frac{\$1800}{\$100 + \$1800} \approx 0.95 = 95\%.$$

Exercise 11.9.26. [S] Can you play?

At `en.wikipedia.org/wiki/Lotteries_in_the_United_States` Wikipedia (probably reliable for these data) says that (as of 2019) 45 states and the District of Columbia have lotteries.

Then

$$\frac{\text{number of states with lotteries}}{\text{number of states}} = \frac{46}{51} = 90\%.$$

(a) Why is it wrong to conclude that there is a 90% probability that a random person chosen from the United States lives in a jurisdiction with a lottery? Is the correct probability more or less than 90%?

(b) How would you compute the probability correctly?

(a) Why is it wrong to conclude that there is a 90% probability that a random person chosen from the United States lives in a jurisdiction with a lottery? Is the correct probability more or less than 90%?

The states with no lottery are Alabama, Alaska, Arizona, Hawaii, Mississippi and Utah, These are much smaller than average (in population) so when you select a random person she will be less likely to live in one of these states.

The correct calculation would give a larger answer. I can see that by imagining just two states, one with 19 people and one with 1. Then calculating by state would give a probability of 1/2 while calculating with populations would give a probability of 19/20.

(b) How would you compute the probability correctly?

$$\frac{\text{number of people living in a jurisdiction with a lottery}}{\text{total population}}.$$

That would be easy in a spreadsheet.

Exercise 11.9.27. [U][Section **??**] Price your raffle tickets.

The website `www.ticketprinting.com/Articles/RaffleTicketPriceCalculator/` provides an online calculator for pricing raffle tickets.

Check that kit does so correctly.

Exercise 11.9.28. [U][N] Savings accounts with benefits.

On January 15, 2014 Tina Rosenberg blogged at *The New York Times* that

> Lotteries aren't usually considered part of the solution to our savings crisis. They're usually cited as a big part of the problem. Lotteries offer

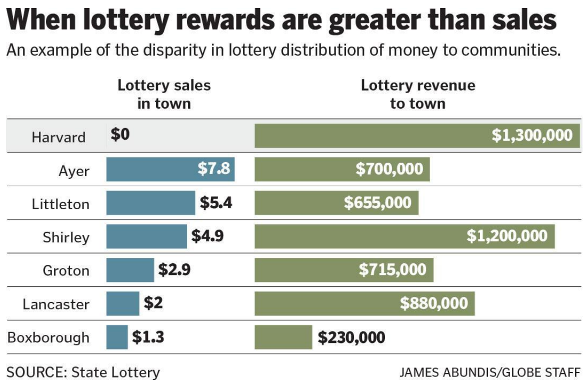


Figure 11.5. Who really wins? [R**??**]

> the worst odds in legal gambling — about 55 percent of what people pay for tickets is paid out in prizes. Yet we spend an average of $540 per household on lottery tickets every year — about $100 more than we spend on milk or beer. That is disproportionately spent by African-Americans, who spend five times as much on lottery tickets per person than whites, and the very poor. People with a household income of less than $10,000 a year who play the lottery spend $597 a year on tickets. [R**??**]

Read further for the savings strategy . . . .

Exercise 11.9.29. [U] Who really wins the lottery?

Figure **??** appeared in *The Boston Globe* on February 25, 2017.

(a) The graphic does not show the units for the dollars in the sales column. Figure out what the $7.8 for Ayer represents.

(b) Note that the bars in the sales column are shorter than the bars in the revenue column. What wrong conclusion does this misleading graphic suggest?

(c) Calculate the cost of the lottery in each town in units (dollar of tickets bought) per (dollar of lottery revenue). Why are all but the first of these numbers less than 1?

(d) Why might it be more informative to report the cost of the lottery in units (dollar of lottery revenue) per (dollar of tickets bought)? Do that.

(e) How do those figures compare to the statewide cost? (Look at an earlier exercise where you worked out the fair price of a dollar ticket.)

Exercise 11.9.30. [U][N] Mass Lottery redux.

On August 31, 2017 Yvonne Abraham wrote in *The Boston Globe* that

> We seem to be especially dippy here in Massachusetts, where we buy more lottery tickets per capita than anywhere in the nation — a whopping $746 per person in fiscal 2017. The $5billionwespentongamesofchancein2017returnedabout1$ billion to cities and towns. For every dollar spent on Powerball tickets, Massachusetts collects 42 cents. Its share of Wanczyk's winnings was $24 million. We'd be sunk without those revenues. [R**??**]

(a) Compare these figures with those in previous exercises.

(b) Is it reasonable or correct to say that the state's share of Wanczayk's *winnings* was $24 million?

R?? J. Jacoby, Lottery games online? Scratch that idea, *The Boston Globe*, July 10, 2016, www.bostonglobe.com/opinion/2016/07/10/lottery-games-online-scratch-that-idea/H0j1VLsYDJa5RZqbbPdfzO/story.html (last visited March 30, 2020).

R?? www.census.gov/programs-surveys/state.html (last visited March 17. 2019).

R?? M. Brown, Did We Get Lucky? LendEDU's Lottery Study & Report, lendedu.com, August 31, 2018, lendedu.com/blog/lottery-study-report/ (last visited March 13, 2019).

R?? upload.wikimedia.org/wikipedia/commons/5/5d/13-02-27-spielbank-wiesbaden-by-RalfR-094.jpg (last visited March 5, 2020), (Creative Commons Attribution-ShareAlike License, en.wikipedia.org/wiki/Wikipedia:Text_of_Creative_Commons_Attribution-ShareAlike_3.0_Unported_License , Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is at commons.wikimedia.org/wiki/Commons:GNU_Free_Documentation_License,_version_1.2).

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