

SEGM

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May 21 – naming numbers¹

“For a change, let’s all work together, and, for a change, try to pay attention from time to time rather than rushing excitedly off to try your own ideas as soon as you get them.”

All (reluctantly): “OK.”

“We’ll start by naming the numbers, and choosing a symbol to write for each one. For the first few we have the usual names and symbols:

0 1 2 3 4 5 6 7 8 9

We don’t want to write ‘10’ next, since that uses two of the symbols over again.”

Sam: “How about B?”

“Good idea.² Then we could continue this way

0 1 2 3 4 5 6 7 8 9 B C D E . . .

¹I wanted to introduce duodecimal arithmetic. When I’ve tried this before with other groups of kids (math club at the Manning, Sam once) I discovered that one of the major problems was distinguishing between numbers and their names. Since I’d had occasion earlier with this group to see how excited they were by choosing names for mathematical entities, I started with that.

²I wondered why he started with ‘B’ rather than ‘A’, and was tempted to revise his suggestion so that we could get to standard hexadecimal later on, but decided just to let it run.

But that's boring. Let's make up symbols instead."

Eva: "How about a circle with a dot in it for after 9? We can call it 'cirdot'?"

I write \odot , "sirdot" with an 's' rather than a 'c'. Eva salutes Sir Dot.

Grace: "'Pie' next." She sketches a complex picture of a tall apple pie.

"I think that will take too long to draw each time we need it. Symbols should be easy to write. How about a piece of pie instead?" I draw Δ .

Grace: "Let's call it 'apple', not 'pie'."

Max: "We'll have 'sirpent' next." He draws \diamond .

Sam (picking up on "serpent"): "Then we can have a snake. It can be just **S**."

Next we got a stick figure, "sirbob," and a light bulb, "sirbulb."

We could clearly have gone on happily punning new knight number names. But I wanted to move on to mathematics, so I posed some arithmetic questions to which they rapidly provided the answers:

$$3 + 5 = 8$$

$$6 + 5 = \Delta$$

$$8 + 5 = \mathbf{S}$$

$$\odot + 2 = \diamond$$

Then I asked for

$$\odot + \diamond$$

and it dawned on everyone (particularly Eva) that we would need more names – in fact, names forever, one for each number.

I asked how we managed everyday arithmetic with just ten symbols.

"After 9 we write 10."

"Why do you think we start naming the numbers with two digits after we reach 9?"

"Because we have 10 fingers."

"So if we had 6 fingers on each hand we'd have invented two more single digit symbols before we moved to two digit numbers. Perhaps

$$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ \odot\ \Delta$$

Then how would you write the next number?"

All agree on a '1' followed by a '0'.

"And what would you call it?"

Sam blurts out "ten" and I admonish him: "No ordinary names for the numbers beyond 9, or we'll soon be very confused!"

Grace: "We can call it 'table'."³

Then we know how to keep counting: "table 1, table 2 and so on. What's after "table 9"?"

All: "table sirdot, table apple, two table." I write down

19, 1 \odot , 1 Δ , 20.

Saying the numbers before writing them guaranteed that they'd say "two table" and not "twenty" for 20.⁴

They decide they want to name the whole number system as well as naming the numbers themselves. Someone suggests "SEGM" for "Sam, Eva, Grace, Max" and all agree.

The addition table

It's time to be systematic and write out SEGM's addition table. They each start with an empty table of the right size in their math notebooks and begin to fill it in.

+	0	1	2	3	4	5	6	7	8	9	\odot	Δ
0	0	1	2	3	4	5	6	7	8	9	\odot	Δ
1	1	2	3	4	5	6	7	8	9	\odot	Δ	10
2	2	3	4	5	6	7	8	9	\odot	Δ	10	11
...												

They know that the columns will be the same as the rows. They quickly see the constant diagonals that slope down to the left. Then they busily fill in

³I thought of suggesting "douze", reminiscent of "dozen", but decided not to interrupt whatever they chose to invent.

⁴I did quietly point out the likely etymology of the English "twenty" as "two ten" but no one was paying attention, which was just as well.

the numbers without further thought. I suggested we move on to something more interesting.⁵

The multiplication table

This begins innocently enough. The first two rows are easy. They have fun with the third row when they see the pattern. Most realize quickly that counting by twos rather than multiplying is the faster way to get the next entry.

\times	0	1	2	3	4	5	6	7	8	9	\odot	Δ
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	\odot	Δ
2	0	2	4	6	8	\odot	10	12	14	16	18	$1\odot$
...												

Max says “sirdot is an even number!” I applaud.⁶
 The rows for 3 and 4 are harder, but still show patterns:

\times	0	1	2	3	4	5	6	7	8	9	\odot	Δ
...												
3	0	3	6	9	10	13	16	19	20	23	26	29
4	0	4	8	10	14	18	20	24	28	30	34	38
...												

They find the row for 5 weird and difficult. No pattern at all. Eva said, with surprise and pleasure, counting on her fingers (just ten of them) “This is hard!” But the row for 6 is a reward.

⁵I think they would still have to pause for thought to compute $\odot + \odot$, but decide not to spend more time on the full table now.

⁶Actually, he said apple was an even number, and at the time I didn’t see his error. But the applause is warranted despite the error. What was important was his realization that Δ and \odot were just numbers with the usual properties of numbers, independent of what they were called. He clearly understood “even” as a property divorced from the last digit in decimal representation.

\times	0	1	2	3	4	5	6	7	8	9	\odot	Δ
...												
5	0	5	\odot	13	18	21	26	2Δ	34	39	42	47
6	0	6	10	16	20	26	30	36	40	46	50	56
...												

Looking over Eva's shoulder, I show her that counting by 6's in SEGM is just like counting by 5's in ordinary English.

I ask them to finish the multiplication table for homework.⁷ Here it is with one more row:

\times	0	1	2	3	4	5	6	7	8	9	\odot	Δ
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	\odot	Δ
2	0	2	4	6	8	\odot	10	12	14	16	18	$1\odot$
3	0	3	6	9	10	13	16	19	20	23	26	29
4	0	4	8	10	14	18	20	24	28	30	34	38
5	0	5	\odot	13	18	21	26	2Δ	34	39	42	47
6	0	6	10	16	20	26	30	36	40	46	50	56
7	0	7	12	19	24	2Δ	36	41	48	53	$5\odot$	65
8	0	8	14	20	28	34	40	48	54	60	68	74
9	0	9	16	23	30	39	46	53	60	69	76	83
\odot	0	\odot	19	26	34	42	50	$5\odot$	69	86	94	$\odot 2$
Δ	0	Δ	$1\odot$	29	38	47	56	65	74	83	92	$\odot 0$
10	0	10	20	30	40	50	60	70	80	90	$\odot 0$	$\Delta 0$

June 18 – what next?

Possibilities:

- The number after $\Delta\Delta$
- Counting, buzz, buzz-skinimerink
- Fractions as decimals

⁷I wonder if they've done it.