CS 420
Final Exam Practice Questions

Below are some practice questions for the final exam. There will be questions on the actual exam that do not resemble any of these questions. Other questions on the exam may ask about similar material but be phrased in a different way. Still, doing all of these practice questions will help you prepare for the exam.

1. Let G be the following Chomsky Normal Form grammar.

$$\begin{aligned} S &\rightarrow AB|CA|a \\ A &\rightarrow BC|a|b \\ B &\rightarrow CC|c \\ C &\rightarrow AC|b \end{aligned}$$

Here is a partially filled in table for the algorithm described in Theorem 7.16 to determine if a particular string belongs to $L(G)$.

		j				
		1	2	3	4	5
	1	B	A	$\emptyset$	?	?
i	2	X	A, C	S	B, C	?
	3	X	X	A, S	C	B, C, S
	4	X	X	X	A, C	B, C, S
	5	X	X	X	X	A, C

(a) What is the string w for this table? (For this grammar, you can tell just by looking at the table.)

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(b) What are the three missing entries in the table? Give complete answers.

$table(1, 4)$ = _____

$table(2, 5)$ = _____

$table(1, 5)$ = _____

(c) Is w in $L(G)$? _____

(d) How does your answer to (c) follow from your answer to (b)?

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2. Let A be the language $\{w\#w^R | w \in \{0, 1\}^*\}$.

(a) Give an implementation-level description of a one-tape Turing machine that decides the language A in time $O(n^2)$. Give a brief explanation of why your Turing machine runs in time $O(n^2)$.

(b) Give an implementation-level description of a multi-tape Turing machine that decides A in time $O(n)$. (You do not need to explain why the running time is $O(n)$.)

3. An undirected graph G is called *bipartite* if there is a set of vertices R such that every edge connects a vertex in R with a vertex not in R. Let

$$BIPARTITE = \{\langle G\rangle | G \text{ is a bipartite graph}\}$$

Show that BIPARTITE is in NP.

4. Show that the grammar given in Exercise 2.13 of the text is ambiguous.

5. If A is language, then Problem 1.40 on page 89 of the text gives the definition of another language $NOPREFIX(A)$. Show that if A is decidable, then $NOPREFIX(A)$ is decidable, by completing the following proof.

Let M be a Turing machine that decides A. Then a Turing machine N that decides $NOPREFIX(A)$ is given by

N="On input w ...

6. Which one of the following statements correctly describes the running time of the Turing machine M_2 given in Example 3.7? (No explanation for your answer is required.)

(a) $\square$ M_2 runs in time $O(n^2)$ but not in time $O(n \log n)$.

(b) $\square$ M_2 runs in time $O(n \log n)$ but not in time $O(n)$.

(c) $\square$ M_2 runs in time $O(n)$.

7. Does the Turing machine R given in the proof of Theorem 4.8 run in polynomial time? Explain your answer.