CS 420 Spring 2019
Homework 10

Due: May 1

1. (a) $REJECT_{TM}$ is defined as $\{\langle M, w\rangle | M$ is a Turing machine, and M rejects $w\}$. Prove that $REJECT_{TM}$ is Turing recognizable.
   (b) Show that $REJECT_{TM}$ is undecidable using diagonalization. Your proof should be similar to, but not the same as, the proof that A_{TM} is undecidable.
   (c) Give a second proof that $REJECT_{TM}$ is undecidable by reducing A_{TM} to $REJECT_{TM}$.
   [This will involve some creativity because the technique we used to reduce A_{TM} to $HALT_{TM}$ will not work here.]

2. Let CONTEXT-FREE$_{TM}$ $= \{\langle M\rangle | M$ is a Turing machine and $L(M)$ is context-free$\}$. Prove that CONTEXT-FREE$_{TM}$ is not decidable by reducing A_{TM} to CONTEXT-FREE$_{TM}$.
   [Hint: Use a proof similar to the proof of Theorem 5.3.]

3. Let NONREGULAR$_{TM}$ $= \{\langle M\rangle | M$ is a Turing machine, and $L(M)$ is not a regular language$\}$. Suppose that you want to reduce A_{TM} to NONREGULAR$_{TM}$ by transforming $\langle M, w\rangle$ to $\langle M_2\rangle$. (So if $\langle M, w\rangle$ is in A_{TM}, then $\langle M_2\rangle$ is in NONREGULAR$_{TM}$, and if $\langle M, w\rangle$ is not in A_{TM}, then $\langle M_2\rangle$ is not in NONREGULAR$_{TM}$.)
   (a) Fill in the blanks in the following two statements in a way that states what you have to do to make the reduction work. Make your statements as general as possible. (In both cases you will be writing down something about the behavior of the Turing machine M_2.)
   - If M accepts w, then

     _____.
   - If M does not accept w, then

     _____.

   (b) Give the definition of the desired Turing machine M_2, given M and w.

4. Problem 5.9.

5. Problem 5.14.

6. Problem 5.15.

7. Problem 5.27
   [Hint: This is hard. First show that the emptiness problem for two-dimensional finite automata is undecidable, using computation histories, then reduce the emptiness problem to the equivalence problem.]