Due: April 18
Note: Numbers in square braces refer to the numbering in the second edition of the text.

1. If $A$ is a language, then $SUFFIX(A)$ is the language
   \[ \{ u | vu \in A \text{ for some string } v \} \]
   (a) Prove that if $A$ is decidable, then $SUFFIX(A)$ is Turing recognizable.
   (b) Prove that if $A$ is Turing recognizable, then $SUFFIX(A)$ is Turing recognizable.
   [Since every decidable language is Turing recognizable, this part implies the first part, but since the proof is harder, I made it a separate part.]

2. Apply the method from class that decides $E_{DFA}$ to the following DFA and answer the questions below.
(a) List the states you mark in the order they get marked.

(b) Does the DFA belong to $E_{DFA}$? 

(c) How does your answer to (b) follow from your answer to (a)?

3. Apply the method from class that decides $E_{CFG}$ to the following CFG and answer the questions below.

$$S \rightarrow aTbU|aSTb$$
$$T \rightarrow YaU|bT$$
$$U \rightarrow aYbY|VW$$
$$V \rightarrow aV|bW$$
$$W \rightarrow aW|bV$$
$$X \rightarrow bX|\varepsilon$$
$$Y \rightarrow aY|a|TU$$

(a) List the terminals and variables you mark in the order they get marked. (List each terminal and variable only the first time you mark it. There is more than one possible order.)

(b) Does the CFG belong to $E_{CFG}$? 

(c) How does your answer to (b) follow from your answer to (a)?

4. The language $EQ_{REX}$ is defined as $\{(R,S)|R,S \text{ are a regular expressions and } L(R) = L(S)\}$. Prove that $EQ_{REX}$ is decidable.

5. Let $ALL_{NFA} = \{\langle A \rangle | A \text{ is an NFA and } L(A) = \Sigma^*\}$. Show that $ALL_{NFA}$ is decidable.

6. Let $F = \{\langle A \rangle | A \text{ is a DFA with input alphabet } \{0,1\} \text{ and every string in } L(A) \text{ has an even number of } 0\text{'s}\}$. Prove that $F$ is decidable.

7. Let $K = \{\langle A \rangle | A \text{ is a DFA and } L(A) \text{ does not contain any string with at least as many } a\text{'s as } b\text{'s}\}$. Show that $K$ is decidable. (The solution to Problem 4.25 [4.23] is useful here.)

8. Let $L = \{\langle P \rangle | P \text{ is a PDA and } L(P) \text{ does not contain any even length string}\}$. Prove that $L$ is decidable.
9. Show that the set that consists of all finite sequences of natural numbers is countable. In other words show that
\[ \{ s | \text{for some } n \geq 0 \text{ and natural numbers } a_1, \ldots, a_n, \ s = (a_1, \ldots, a_n) \} \]
is a countable set.

10. An infinite sequence \( a_1a_2\cdots \) of natural numbers is called \textit{strictly increasing} if \( a_1 < a_2 < a_3 < \cdots \). Let \( B \) be the set of all strictly increasing sequences of natural numbers. Use diagonalization to prove that \( B \) is uncountable.