1. Let $F = \{ \langle A \rangle | A$ is a DFA with input alphabet $\{0, 1\}$ and some string in $L(A)$ contains 011 as a substring $\}$. Prove that $F$ is decidable.

2. Let $K = \{ \langle A \rangle | A$ is a DFA with alphabet $\{a, b\}$ and $L(A)$ does not contain any string with exactly two more $a$'s than $b$'s $\}$. Show that $K$ is decidable. (The solution to Problem 4.25 [4.23] is useful here.)

3. Let $L = \{ \langle P \rangle | P$ is a PDA with input alphabet $\{0, 1\}$ and no string in $L(P)$ contains 011 as a substring $\}$. Prove that $L$ is decidable.

4. Show that the set that consists of all finite sequences of natural numbers is countably infinite. In other words show that

\[
\{ s | \text{for some } n \geq 0 \text{ and natural numbers } a_1, \ldots, a_n, s = \langle a_1, \ldots, a_n \rangle \}
\]

is a countably infinite set.

5. An function $f : \mathbb{N} \to \mathbb{N}$ is called strictly increasing if $f(1) < f(2) < f(3) < \cdots$. Let $B$ be the set of all strictly increasing functions from $\mathbb{N}$ to $\mathbb{N}$. Use diagonalization to prove that $B$ is uncountable.

6. Let $C$ be the set of infinite binary sequences $a_1a_2a_3\cdots$ such that $a_1 = a_3 = a_5 = \cdots = 0$. In other words a sequence in $C$ can have either 0 or 1 in the even positions, but has to have 0 in the odd positions. Prove that $C$ is uncountable.