Due: April 30

Note: This homework is long because it covers the material for three classes, but these problems are relevant for the test. Please look at them even if you don’t have time to work hard on all of them.

1. Let CONTEXT-FREE$TM = \{\langle M \rangle | M$ is a Turing machine and $L(M)$ is context-free$\}$. Prove that CONTEXT-FREE$TM$ is not decidable. [Hint: Use a proof similar to the proof of Theorem 5.3.]

2. Let NONREGULAR$TM = \{\langle M \rangle | M$ is a Turing machine, and $L(M)$ is not a regular language$\}$. Suppose that you want to reduce $A_{TM}$ to NONREGULAR$TM$ by transforming $\langle M, w \rangle$ to $\langle M_2 \rangle$. (So if $\langle M, w \rangle$ is in $A_{TM}$, then $\langle M_2 \rangle$ is in NONREGULAR$TM$, and if $\langle M, w \rangle$ is not in $A_{TM}$, then $\langle M_2 \rangle$ is not in NONREGULAR$TM$.)

   (a) Fill in the blanks in the following two statements in a way that states what you have to do to make the reduction work. Make your statements as general as possible. (In both cases you will be writing down something about the behavior of the Turing machine $M_2$.)

   • If $M$ accepts $w$, then

   • If $M$ does not accept $w$, then

   (b) Give the definition of the desired Turing machine $M_2$, given $M$ and $w$.

3. Problem 5.12.

4. Problem 5.15.

5. Problem 5.27
   [Hint: This is hard. First show that the emptiness problem for two-dimensional finite automata is undecidable, using computation histories, then reduce the emptiness problem to the equivalence problem.]

6. Exercise 5.4.

7. Problem 5.22.

8. Problem 5.23.


10. Problem 5.25.
11. Is it possible to $m$-reduce $E_{LBA}$ to $A_{LBA}$? Explain your answer.

12. Is it possible to $m$-reduce $EQ_{TM}$ to $\overline{A_{TM}}$? Explain your answer.