1. Give DFAs that recognize the following languages

(a) \( \{w \in \{0,1\}^* | w \text{ starts with a 0 and has odd length} \} \).

Solution:

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(a) {w ∈ {0,1}∗|w starts with a 0 and has odd length}.
```

(b) \( \{w \in \{0,1\}^* | w \text{ contains 011 as a substring} \} \).

Solution:

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(b) {w ∈ {0,1}∗|w contains 011 as a substring}.
```
(c) \(\{00.001\}^*\).

Solution:

2. Using the complementation construction and one of the DFAs from Exercise 1, give a DFA that recognizes the language

\(\{w \in \{0,1\}^*|w \text{ does not contain 011 as a substring}\}\).

Solution:
3. Let \( L_1 = \{0^n | n \text{ is divisible by } 2 \} \) and \( L_2 = \{0^n | n \text{ is divisible by } 3 \} \). In class, we gave a two-state DFA \( M_1 \) that recognizes \( L_1 \) and a three-state DFA \( M_2 \) that recognizes \( L_2 \). Using the union construction of Theorem 1.25, combine the \( M_1 \) and \( M_2 \) to obtain a DFA that recognizes \( L = \{0^n | n \text{ is divisible by } 2 \text{ or } 3 \} \). How does \( M \) compare with the DFA we gave in class that recognizes \( L \)?

**Solution:**

The DFAs \( M_1 \) and \( M_2 \) are:

\[
\begin{align*}
M_1: & \quad \begin{array}{c}
0 \quad 0 \\
\downarrow \quad \downarrow \\
0 \quad 1
\end{array} \\
M_2: & \quad \begin{array}{c}
2 \quad \downarrow \\
\downarrow \\
\quad 0
\end{array} \\
& \quad \begin{array}{c}
3 \quad \downarrow \\
\downarrow \\
\quad 0
\end{array} \\
& \quad \begin{array}{c}
4 \quad \downarrow \\
\downarrow \\
\quad 0
\end{array}
\end{align*}
\]

The union construction gives the following DFA when unreachable states are removed:

\[
\begin{align*}
(0,2) & \quad \begin{array}{c}
0 \quad 0 \\
\downarrow \quad \downarrow \\
(0,3) \quad (1,4)
\end{array} \\
(1,3) & \quad \begin{array}{c}
0 \quad \downarrow \\
\downarrow \\
(0,4) \quad (1,2)
\end{array}
\end{align*}
\]

This is the same as the DFA we gave in class.
4. Let $L = \{w \in \{0, 1\}^* | w \text{ starts with a 0 and has even length}\}$. Starting with DFAs for two simpler languages, use the intersection construction to give a DFA that recognizes $L$.

**Solution:**

Let $L_1 = \{w \in \{0, 1\}^* | w \text{ starts with a 0}\}$ and $L_2 = \{w \in \{0, 1\}^* | w \text{ has even length}\}$. Then, $L = L_1 \cap L_2$. A DFA recognizing $L_1$ is given by

![DFA for $L_1$]

and a DFA recognizing $L_2$ is given by

![DFA for $L_2$]

The intersection construction gives the following DFA that recognizes $L_1 \cap L_2 = L$. 

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5. (a) Give an NFA with four states that recognizes $L_1 = \{w \in \{0, 1\}^* | \text{one of the last three symbols in } w \text{ is a 1} \}$.
   
   Solution:

   (b) Give an NFA with three states that recognizes the language $L_2 = 0^*1^* \cup 0^*1^*2$.
6. Convert the NFA given in Figure 1.27 of the textbook into a DFA. Show only the reachable states of the DFA.

Solution: