1. Give DFAs that recognize the following languages

(a) \( \{ w \in \{0,1\}^* | w \text{ contains exactly three 1's} \} \).

Solution:

```
       0  0  0
        ↑  ↑  ↑
         0  1  1
         0  1  1
```

(b) \( \{ w \in \{0,1\}^* | w \text{ contains 001 as a substring} \} \).

Solution:

```
       0
        ↑
         0
         0
         0,1
```

```
(c) \(\{11,110\}^*\).

Solution:

\[
\begin{array}{c}
\text{0} \\
\text{1} \\
\text{0, 1}
\end{array}
\]

2. Using the complementation construction and one of the DFAs from Exercise 1, give a DFA that recognizes the language

\[\{w \in \{0,1\}^* | w \text{ does not contain 001 as a substring}\}.

Solution:

\[
\begin{array}{c}
\text{1} \\
\text{0} \\
\text{0} \\
\text{0, 1}
\end{array}
\]

3. Let \(L = \{w \in \{0,1\}^* | w \text{ ends with a 0 or has even length}\}\). Starting with DFAs for two simpler languages, use the union construction to give a DFA that recognizes \(L\).

Solution:

Let \(L_1 = \{w \in \{0,1\}^* | w \text{ ends with a 0}\}\) and \(L_2 = \{w \in \{0,1\}^* | w \text{ has even length}\}\). Then, \(L = L_1 \cup L_2\). A DFA recognizing \(L_1\) is given by
and a DFA recognizing $L_2$ is given by

The union construction gives the following DFA that recognizes $L_1 \cup L_2 = L$. 