1. Give DFAs that recognize the following languages

(a) \(\{w \in \{0, 1\}^* | w \text{ contains at least two 0's}\}\).

Solution:

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1 --0-> 2 --0-> 3
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(b) \(\{w \in \{0, 1\}^* | w \text{ contains 110 as a substring}\}\).

Solution:

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4 --1-> 5 --1-> 6 --0-> 7
```
2. Using the complementation construction and one of the DFAs from Exercise 1, give a DFA that recognizes the language
\( \{w \in \{0, 1\}^* | w \text{ does not contain } 110 \text{ as a substring}\} \).

**Solution:**

3. Let \( A \) be the language \( \{0^n1^n | n \geq 0\} \). What is wrong with the following “proof” that \( A \) is regular?

**Proof:**

Consider the DFA given below
The DFA accepts every string in \( A \), so \( A \) is regular.

**Solution:** In order for a language \( A \) to be regular, there must be a DFA \( M \) that recognizes \( A \). \( M \) recognizing \( A \) means that

1. \( M \) accepts every string in \( A \), and
2. \( M \) rejects every string not in \( A \).

For the DFA \( M \) given in the “proof,” \( M \) accepts every string in \( A \), but \( M \) does not reject every string not in \( A \). For instance, \( M \) accepts the string 0 even though 0 is not in \( A \). Thus, \( M \) does not recognize \( A \). The fact that \( M \) accepts every string in \( A \) is not sufficient to make \( A \) regular; \( M \) would also have to reject all strings not in \( A \), and that is not true here.