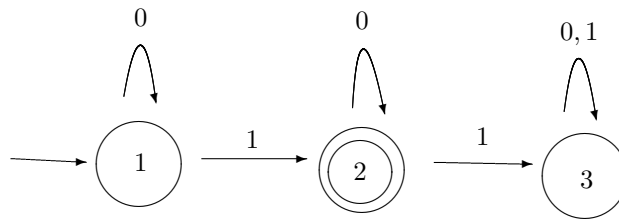


**CS 420, Spring 2019**  
**Homework 2 Solutions**

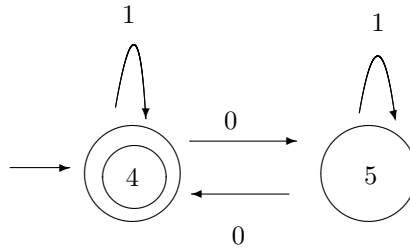
- Let  $L = \{w \in \{0,1\}^* \mid w \text{ contains exactly one 1 and an even number of 0's}\}$ . Starting with DFAs for two simpler languages, use the intersection construction to give a DFA that recognizes  $L$ .

**Solution:**

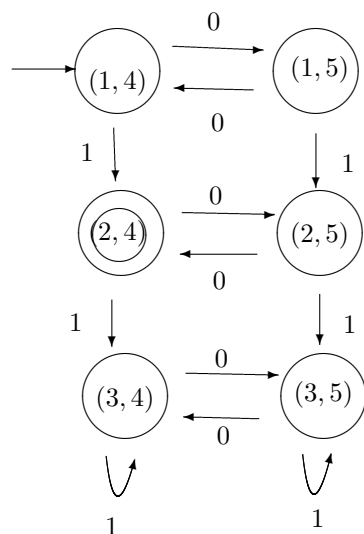
Let  $L_1 = \{w \in \{0,1\}^* \mid w \text{ contains exactly one 1}\}$  and  $L_2 = \{w \in \{0,1\}^* \mid w \text{ contains an even number of 0's}\}$ . Then,  $L = L_1 \cap L_2$ . A DFA recognizing  $L_1$  is given by



and a DFA recognizing  $L_2$  is given by

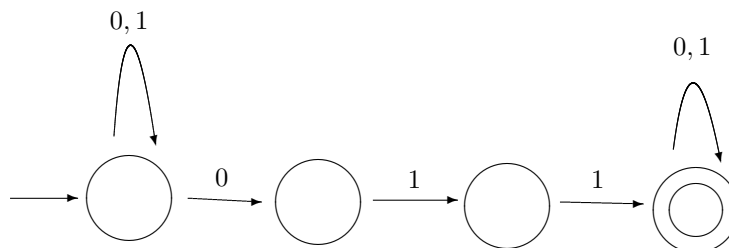


The intersection construction gives the following DFA that recognizes  $L_1 \cap L_2 = L$ .



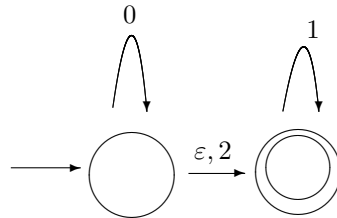
2. (a) Give an NFA with four states that recognizes  $L_1 = \{w \in \{0,1\}^* | w \text{ contains } 011 \text{ as a substring}\}$ .  
 [You can give a DFA with four states that recognizes  $L_1$ , but you should use nondeterminism to give an NFA that is simpler than the DFA.]

**Solution:**



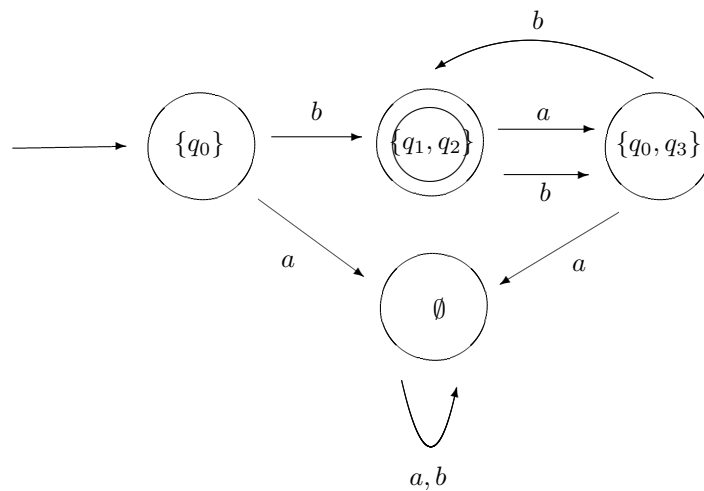
- (b) Give an NFA with two states and only one accept state that recognizes the language  $L_2 = 0^*1^* \cup 0^*21^*$ .

**Solution:**



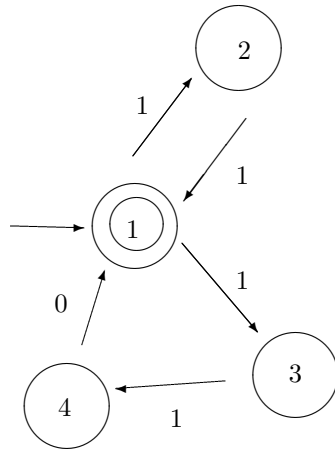
3. Convert the NFA given in Slide 91 of the slides into a DFA. Show only the reachable states of the DFA.

**Solution:**



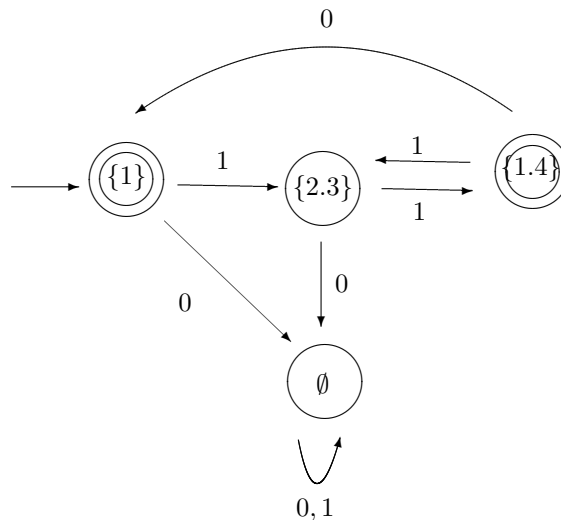
4. Let  $L = \{11, 110\}^*$ .
- (a) Give an NFA  $N$  with four states that recognizes  $L$ . Your NFA should be similar to the NFA we gave in class to recognize  $\{01, 010\}^*$ .

**Solution:**



(b) Using the method from class, convert  $N$  to a DFA  $M$ .

**Solution:** (We show only the reachable states of  $M$ .)

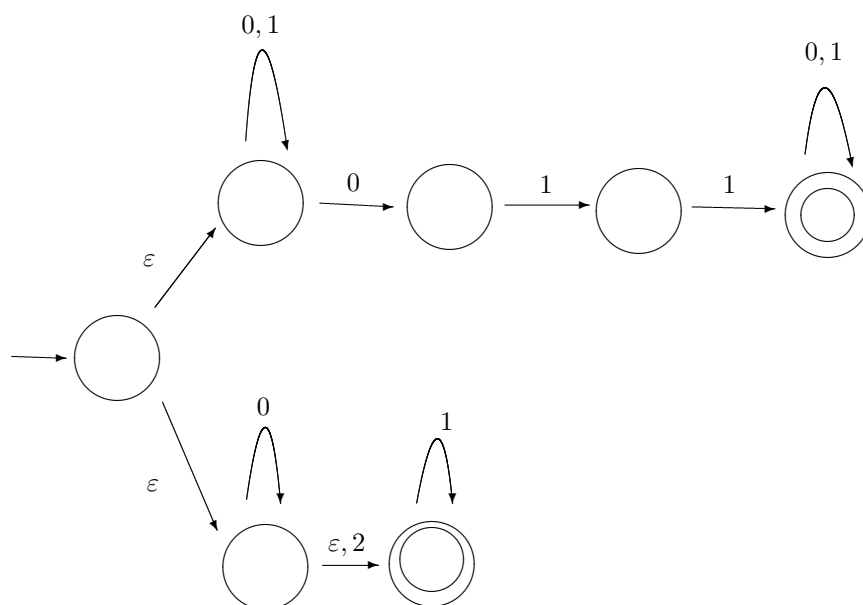


(c) How does  $M$  compare with the DFA given in the solutions to Exercise 1c of Homework 1?

**Solution:** The full DFA  $M$  has 16 states, so is not the same as the DFA in the solutions to Exercise 1c of Homework 1, but once unreachable states are removed from  $M$ , we get the same DFA as in the solutions.

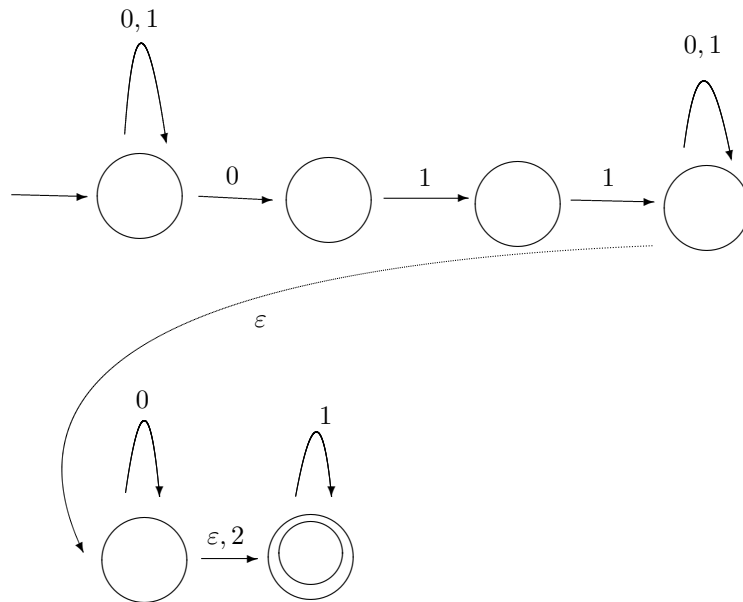
5. Using the method from class, give an NFA that recognizes  $L_1 \cup L_2$ , where  $L_1$  and  $L_2$  are the languages from Exercise 2.

**Solution:**



6. Using the method from class, give an NFA that recognizes  $L_1 \circ L_2$ , where  $L_1$  and  $L_2$  are the languages from Exercise 2.

**Solution:**



7. Using the method from class, give an NFA that recognizes  $L_2^*$ , where  $L_2$  is the language from Exercise 2.

**Solution:**

