1. Using the union construction and two of the DFAs from Exercise 1 of Homework 1, give a DFA that recognizes the language

\[ \{ w \in \{0, 1\}^* | w \text{ ends with 011 or is in } \{01, 0100\}^* \}. \]

(Your answer must be a DFA obtained using the construction from Theorem 1.25 and not just an NFA.)

**Solution:**

![DFA Diagram]
2. In Exercise 1 of Homework 1, you were asked to give a DFA for \( \{ w \in \{0,1\}^* | w \text{ starts with a 1 and has even length} \} \). Show how to get a DFA for this language using the intersection construction and DFAs for two simpler languages.

**Solution:**

The language can be expressed as \( L_1 \cap L_2 \), where \( L_1 = \{ w \in \{0,1\}^* | w \text{ starts with a 1} \} \) and \( L_2 = \{ w \in \{0,1\}^* | w \text{ has even length} \} \).

A DFA for \( L_1 \) is given by

![DFA for L1](image1)

and a DFA for \( L_2 \) is given by

![DFA for L2](image2)

Using the intersection construction, we get the following DFA that recognizes \( \{ w \in \{0,1\}^* | w \text{ starts with a 1 and has even length} \} \).

![Intersection DFA](image3)
3. (a) Give an NFA with four states that recognizes $L_1 = \{ w \in \{0, 1\}^* | w$ has length at least 3 and the third symbol from the right in $w$ is a 1}.  

**Solution:**

```plaintext
0, 1

```

(b) Give an NFA with three states that recognizes the language $L_2 = 1^*2^* \cup 01^*2^*$.  

**Solution:**

```plaintext
\varepsilon, 0

1

\varepsilon

2

```
4. Convert the NFA given in Slide 91 of the slides into a DFA. Show only the reachable states of the DFA.

Solution:

\[
\begin{align*}
\{q_0\} \xrightarrow{a} \{q_0, q_3\} \xrightarrow{b} \{q_1, q_2\} \xrightarrow{a} \{q_0, q_3\} \\
\emptyset \xrightarrow{a, b} \{q_0\}
\end{align*}
\]

5. Using the method from class, give an NFA that recognizes \(L_1 \cup L_2\), where \(L_1\) and \(L_2\) are the languages from Exercise 3.

Solution:
6. Using the method from class, give an NFA that recognizes \( L_1 \circ L_2 \), where \( L_1 \) and \( L_2 \) are the languages from Exercise 3.

Solution:

\[
\begin{array}{c}
\text{0, 1} \\
\rightarrow 1 \\
\rightarrow 0, 1 \\
\rightarrow 0, 1 \\
\rightarrow \epsilon \\
\rightarrow \epsilon, 0 \\
\rightarrow \epsilon \\
\rightarrow \epsilon \\
\rightarrow \text{\bullet}
\end{array}
\]