1. Let $L = \{w \in \{0,1\}^* | w \text{ contains exactly one 1 and an even number of 0's} \}$. Starting with DFAs for two simpler languages, use the intersection construction to give a DFA that recognizes $L$.

Solution:
Let $L_1 = \{w \in \{0,1\}^* | w \text{ contains exactly one 1} \}$ and $L_2 = \{w \in \{0,1\}^* | w \text{ contains an even number of 0's} \}$. Then, $L = L_1 \cap L_2$. A DFA recognizing $L_1$ is given by

![Diagram 1](image1.png)

and a DFA recognizing $L_2$ is given by

![Diagram 2](image2.png)

The intersection construction gives the following DFA that recognizes $L_1 \cap L_2 = L$. 

1
2. (a) Give an NFA with four states that recognizes $L_1 = \{ w \in \{0, 1\}^* | w$ contains 011 as a substring $\}$. [You can give a DFA with four states that recognizes $L_1$, but you should use nondeterminism to give an NFA that is simpler than the DFA.]

Solution:

(b) Give an NFA with two states and only one accept state that recognizes the language $L_2 = 0^*1^* \cup 0^*21^*$. 

Solution:
3. Convert the NFA given in Slide 91 of the slides into a DFA. Show only the reachable states of the DFA.

Solution:

4. Let $L = \{11, 110\}^*$.

(a) Give an NFA $N$ with four states that recognizes $L$. Your NFA should be similar to the NFA we gave in class to recognize $\{01, 010\}^*$.

Solution:
(b) Using the method from class, convert $N$ to a DFA $M$.

**Solution:** (We show only the reachable states of $M$.)

(c) How does $M$ compare with the DFA given in the solutions to Exercise 1c of Homework 1?

**Solution:** The full DFA $M$ has 16 states, so is not the same as the DFA in the solutions to Exercise 1c of Homework 1, but once unreachable states are removed from $M$, we get the same DFA as in the solutions.

5. Using the method from class, give an NFA that recognizes $L_1 \cup L_2$, where $L_1$ and $L_2$ are the languages from Exercise 2.
6. Using the method from class, give an NFA that recognizes $L_1 \circ L_2$, where $L_1$ and $L_2$ are the languages from Exercise 2.

Solution:
7. Using the method from class, give an NFA that recognizes $L_2^*$, where $L_2$ is the language from Exercise 2.

Solution: