1. Using the method from class, give an NFA that recognizes $L_2^*$, where $L_2$ is the language from Exercise 3(b) of Homework 2.

**Solution:**

![NFA Diagram]

2. Give regular expressions for the following languages:

   (a) $\{w \in \{0, 1\}^* | w$ ends with either 110 or 01$\}$
   
   **Solution:** $(0 \cup 1)^*(110 \cup 01)$

   (b) $\{w \in \{0, 1\}^* | w$ starts with a 1 and has even length$\}$
   
   **Solution:** $1(0 \cup 1)((0 \cup 1)(0 \cup 1))^*$

   (c) $\{w \in \{0, 1\}^* | w$ has length at least 3 and the third symbol from the right in $w$ is a 1$\}.$
   
   **Solution:** $(0 \cup 1)^*1((0 \cup 1)(0 \cup 1))$

   (d) $\{w \in \{0, 1\}^* | w$ does not end with 110 and does not end with 01$\}$.
   
   **Solution:** $\varepsilon \cup 0 \cup 1 \cup 00 \cup 10 \cup 11 \cup (0 \cup 1)^*(000 \cup 01(0 \cup 1) \cup 100 \cup 111)$

3. Convert the regular expression $(a \cup b)^*c$ into an NFA using the method from class (which is the same as the method from the book and is different from the method in JFLAP).

**Solution:**

First we have the following NFAs for $a$, $b$, and $c$. 

[Diagram for a, b, and c]
Then, we obtain an NFA for $a \cup b$

![Diagram](image1)

and the following for $(a \cup b)^*$.  

![Diagram](image2)

Finally, we obtain the NFA below for $(a \cup b)^*c$.

![Diagram](image3)
4. Let $N$ be the following NFA:

Convert $N$ into a regular expression using the method from class (which is the same as the method in the book, and is not the same as the method in JFLAP).

**Solution:**

Turning this into a GNFA, we get
Eliminating state $q_4$ gives

Eliminating state $q_0$ gives
Eliminating state $q_2$ gives
Eliminating state $q_1$ gives

Finally, eliminating state $q_3$ gives

and the regular expression is $(a^*b\cup a^*(b \cup ba^*ab))(aa^*ab \cup b^*(b \cup ba^*ab))^*$.

5. **Problem 1.31** Show that if $A$ is regular so is $A^R$.

**Proof:** Since $A$ is regular, there is some NFA $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes $A$. We construct an NFA $M'$ such that $L(M') = A^R$. The idea is that the start state of $M'$ is the accept state of $M$, the accept state
of $M'$ is the start state of $M$, and for every transition in $M$, there is a transition with the same label in $M'$, but going in the reverse direction.

The problem with this is that $M$ may have more than one accept state. Since $M'$ can have only one start state, we add a new start state to $M'$ and $\varepsilon$ transitions from the new start state to all the accept states of $M$.

The formal construction is:

$$M' = (Q \cup \{q_s\}, \Sigma, \delta', q_s, \{q_0\})$$

where $q_s$ is a new state and

$$\delta'(q, a) = \begin{cases} 
\{p : q \in \delta(p, a)\} & \text{if } q \neq q_s \\
F & \text{if } q = q_s, a = \varepsilon \\
\emptyset & \text{if } q = q_s, a \neq \varepsilon
\end{cases}$$

6. (a) Let $M$ be the NFA given in the solution to Problem 3(b) on Homework 2. Give an NFA $N$ with three states and no $\varepsilon$-transitions that recognizes the same language.

**Solution:**

![Diagram of NFA with three states and no \(\varepsilon\)-transitions]

(b) Generalize what you did in Part (a) of this problem by proving the following theorem:

**Theorem:** If $M$ is an NFA, then there is an NFA $N$ with the following properties

1. $N$ has the same number of states as $M$.
2. $N$ has no $\varepsilon$-transitions.
3. $L(N) = L(M)$.

**Solution:** We obtain $N$ from $M$ by doing three things:

1. We remove all $\varepsilon$-transitions.
2. If in $M$ there is a series of $\varepsilon$-transitions that leads from a state $p$ to an accept state $q$, then in $N$, $p$ is also an accept state.
3. If in $M$ there is a series of $\varepsilon$-transitions from a state $p$ to a state $q$, and $M$ can go from $q$ to a state $r$ reading a symbol $a$, then $N$ can go directly from $p$ to $r$ reading $a$.

Formally, let $M = (Q, \Sigma, \delta, q_0, F)$. We define $N = (Q, \Sigma, \delta', q_s, F')$.

If $q$ is a state in $Q$, we use the notation $E_M(q)$ for the set of states...
that can be reached from $q$ using 0 or more $\varepsilon$-transitions in $M$. The definition of $N$ is now given by

$$F' = \{q \in Q | E_M(q) \cap F \neq \emptyset\}$$

and

$$\delta'(q, a) = \begin{cases} 
\emptyset & \text{if } a = \varepsilon \\
\bigcup \{\delta(p, a) | p \in E_M(q)\} & \text{if } a \neq \varepsilon 
\end{cases}$$