1. Using the method from class, give an NFA that recognizes $L_2^*$, where $L_2$ is the language from Exercise 3(b) of Homework 2.

**Solution:**

```
\[ \begin{array}{c}
\circ \rightarrow \varepsilon \\
\varepsilon \rightarrow \varepsilon, 0 \\
\varepsilon \rightarrow \varepsilon \\
\varepsilon \rightarrow \varepsilon
\end{array} \]
```

2. Give regular expressions for the following languages:

(a) \{w \in \{0, 1\}^* | w ends with either 110 or 01\}

**Solution:** \((0 \cup 1)^*(110 \cup 01)\)

(b) \{w \in \{0, 1\}^* | w starts with a 1 and has even length\}

**Solution:** \(1(0 \cup 1)((0 \cup 1)(0 \cup 1))^*\)

(c) \{w \in \{0, 1\}^* | w has length at least 3 and the third symbol from the right in w is a 1\}

**Solution:** \((0 \cup 1)^*1((0 \cup 1)(0 \cup 1))\)

(d) \{w \in \{0, 1\}^* | w does not end with 110 and does not end with 01\}

**Solution:** \(\varepsilon \cup 0 \cup 1 \cup 00 \cup 10 \cup 11 \cup (0 \cup 1)^*(000 \cup 01(0 \cup 1) \cup 100 \cup 111)\)

3. Convert the regular expression \((a \cup b)^*c\) into an NFA using the method from class (which is the same as the method from the book and is different from the method in JFLAP).

**Solution:**

First we have the following NFAs for \(a\), \(b\), and \(c\).
Then, we obtain an NFA for \( a \cup b \)

![Diagram for \( a \cup b \)](image)

and the following for \((a \cup b)^*\).

![Diagram for \((a \cup b)^*\)](image)

Finally, we obtain the NFA below for \((a \cup b)^*c\). 

![Diagram for \((a \cup b)^*c\)](image)
4. Let $N$ be the following NFA:

Convert $N$ into a regular expression using the method from class (which is the same as the method in the book, and is not the same as the method in JFLAP).

**Solution:**

Turning this into a GNFA, we get
Eliminating state $q_4$ gives

Eliminating state $q_0$ gives
Eliminating state $q_2$ gives
Eliminating state $q_1$ gives

Finally, eliminating state $q_3$ gives

and the regular expression is $(a^*b \cup a^*b^*(b \cup ba^*ab))(aa^*ab \cup b^*(b \cup ba^*ab))^*$.

5. **Problem 1.31** Show that if $A$ is regular so is $A^R$.

**Proof:** Since $A$ is regular, there is some NFA $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes $A$. We construct an NFA $M'$ such that $L(M') = A^R$. The idea is that the start state of $M'$ is the accept state of $M$, the accept state
of \( M' \) is the start state of \( M \), and for every transition in \( M \), there is a transition with the same label in \( M' \), but going in the reverse direction. The problem with this is that \( M \) may have more than one accept state. Since \( M' \) can have only one start state, we add a new start state to \( M' \) and \( \varepsilon \) transitions from the new start state to all the accept states of \( M \).

The formal construction is:

\[
M' = (Q \cup \{q_s\}, \Sigma, \delta', q_s, \{q_0\})
\]

where \( q_s \) is a new state and

\[
\delta'(q, a) = \begin{cases} 
\{ p : q \in \delta(p, a) \} & \text{if } q \neq q_s \\
F & \text{if } q = q_s, a = \varepsilon \\
\emptyset & \text{if } q = q_s, a \neq \varepsilon 
\end{cases}
\]