CS 420 Spring 2019
Homework 4

Due February 27

1. Let A be the language consisting of those strings w in $\{0, 1, \#\}^*$ such that either w starts with 0 or $w = u\#u$ for some $u \in \{0, 1\}^*$. A is not regular. In a Pumping Lemma proof of this, you are given p and you choose s. For each of the following possible choices of s, state whether or not the choice is a good one. If the choice is bad, provide the decomposition that allows the string to be pumped.
   (a) $s = 0^p1^p\#0^p1^p$;
   (b) $s = 1^p0^p\#1^p0^p$;
   (c) $s = (10)^p\#(10)^p$.

2. Read the discussion of minimum pumping length given in Problem 1.55 of the text (third US edition) and then give the minimum pumping length for the following languages. Justify your answers.
   (a) $\{w \in \{0, 1\}^* | w \text{ contains exactly three 1's}\}$.
   (b) $\{w \in \{0, 1\}^* | w \text{ contains 001 as a substring}\}$.
   (c) $\{11, 110\}^*$.

3. The Pumping Lemma says that every regular language has the Pumping Property. This means that if a language does not have the Pumping Property, then it is not regular. We said in class many times that if a language does have the Pumping Property, then it may or may not be regular. The purpose of this exercise is to give an example of a language that has the Pumping Property but is not regular, which shows that a language with the Pumping Poperty may or may not be regular.

   Define

   $L = \{a^iw | i \geq 0, w \in \{b, c\}^*$ and if $i = 1$, then w has the same number of b's and c's$\}$

   In other words, if $i \neq 1$, then a^iw is in L no matter what $w \in \{b, c\}^*$ is, but if $i = 1$, then w must have the same number of b's as c's in order for a^iw to be in L.
   (a) Show that L has the Pumping Property.
   (b) Show that L^R does not have the Pumping Propertry, so is not regular.
   (c) Use the result in Homework 3, Exercise 3 to show that L is not regular.

4. Give context-free grammars for the following languages:
   (a) $\{0^n1^n2^m3^m | n, m \geq 0\}$.

(b) $\{x\#y | x, y \in \{0,1\}^*$ and $|x| = 2|y|\}$.

(c) $\{x\#y | x, y \in \{0,1\}^*$ and $|x| \neq 2|y|\}$.