Due October 5

Homework 4

1. Let $A$ be the language consisting of those strings $w$ in $\{0, 1, \#\}^\ast$ such that either $w$ starts with 0 or $w = u\#u$ for some $u \in \{0, 1\}^\ast$. $A$ is not regular. In a Pumping Lemma proof of this, you are given $p$ and you choose $s$. For each of the following possible choices of $s$, state whether or not the choice is a good one. If the choice is bad, provide the decomposition that allows the string to be pumped.

(a) $s = 0^p1^p\#0^p1^p$;
(b) $s = 1^p0^p\#1^p0^p$;
(c) $s = (10)^p\#(10)^p$.

2. Read the discussion of minimum pumping length given in Problem 1.55 of the text (third US edition) and then give the minimum pumping length for the following languages. Justify your answers.

(a) $0^\ast1^\ast0^\ast$;
(b) $\{0^n10^m|n + m \text{ is divisible by } 3\}$;
   [This one is hard.]
(c) $\{w \in \{0, 1\}^\ast|w| \leq 5\}$.

3. Problem 1.54. (3rd edition)

4. Give context-free grammars for the following languages:

(a) $\{0^n12^n|n \geq 0\}$;
(b) $\{0^n1^m|n < m\}$
(c) $\{w\#u|w, u \in \{0, 1\}^\ast\text{ and }|w| > 2|u|\}$;
(d) $\{x_1\#x_2\#x_3|x_1, x_2, x_3 \in \{a, b\}^\ast\text{ and either }x_3 = x_1^R\text{ or }x_3 = x_2^R\}$.

5. In class, we gave the following two grammars for the language $L = \{w|w \text{ contains the same number of } a\text{'s and } b\text{'s}\}$.

\[
S \rightarrow \varepsilon|aB|bA \\
A \rightarrow aS|bAA \\
B \rightarrow bS|aBB
\]

and

\[
S \rightarrow aS|bS|SS|\varepsilon
\]

For each of these grammars, give both a leftmost derivation and a parse tree for the string $aababbab$. 

1
6. Show that the first grammar given in the previous problem (the one with three variables) is ambiguous.

7. (a) Give a context-free grammar that generates the language

\[ A = \{ w \in \{a, b\}^* | w \text{ has more } a\text{'s than } b\text{'s} \} \]

[Hint: In class, we gave two grammars that generate the set of strings with the same number of a’s as b’s. You can use either one of these grammars as part of the grammar you give.]

(b) Give a justification that your grammar is correct. Use the same type of argument as we gave in class that the grammars that generate strings with the same number of a’s and b’s are correct. You do not need to reprove the correctness of these grammars.

8. Give an unambiguous grammar for the language \( L \) of Problem 5.

(This is a difficult problem, but give it a try. As a hint, you can use two variables other than the start symbol. One variable generates strings with the same number of a’s as b’s that have the additional property that every prefix has at least as many a’s as b’s, and the second variable generates all strings with the same number of a’s as b’s that have the additional property that every prefix has at least as many b’s as a’s.)