CS 420 Spring 2019
Homework 5

Due: March 6

1. Give regular expressions for the following languages: [A problem of this type should have been on Homework 3, but I forgot to do this.]

   (a) $\{w \in \{0,1\}^* | w$ contains exactly three 1's$\}$

   (b) $\{w \in \{0,1\}^* | w$ contains either 001 or 100 as a substring$\}$

   (c) $\{w \in \{0,1\}^* | w$ has length at least 3 and the third symbol from the right in w is a 0$\}$.

   (d) $\{w \in \{0,1\}^* | w$ does not contain 001 as a substring$\}$.
   [This one is tricky, but there is a short regular expression for this language.]

2. Let L be the language $\{w \in \{a,b\}^* | w$ contains exactly one more b than $a\}$.

   (a) Give a context-free grammar that generates L.

   (b) Give a leftmost derivation and a parse tree in your grammar for the string *abbabab*.

3. Give an unambiguous grammar for the language L of the previous problem.

   (This is a difficult problem, but give it a try. As a hint, you can use three variables other than the start symbol. One variable generates strings with the same number of a's as b's, the second variable generates strings with the same number of a's as b's that have the additional property that every prefix has at least as many a's as b's, and the third variable generates all strings with the same number of a's as b's that have the additional property that every prefix has at least as many b's as a's.)

4. Let A be the language $\{a^n b^n | n \geq 0\}$ and let $B = \overline{A}$.

   Using closure of the context-free languages under union, give a context-free grammar that generates B.
   [Hint: You can express B as the union of three languages, one of which is $\overline{a^* b^*}$.]

5. Let C be the language

   $$\{0^n 1^m 2^p 3^q | n, m, p, q \geq 0 \text{ and } n > m \text{ and } p < q\}.$$

   Using closure of the context-free languages under concatenation, give a context-free grammar for C.

6. Using the method from class, convert the regular expression $(a \cup \varepsilon) b^*$ into a context-free grammar.

7. Give right regular grammars for the following languages:

   (a) $\{w \in \{0,1\}^* | w$ contains exactly three 1's$\}$

   (b) $\{w \in \{0,1\}^* | w$ contains either 001 or 100 as a substring$\}$

   (c) $\{w \in \{0,1\}^* | w$ has length at least 3 and the third symbol from the right in w is a 0$\}$.

   (d) $\{w \in \{0,1\}^* | w$ does not contain 001 as a substring$\}$.

8. Using the method from class, convert the DFA given in the solutions to Problem 1c on Homework 1 into a right regular grammar.

9. Convert the following right regular grammar into an NFA.

$$\begin{array}{rcl} S & \rightarrow & 0S|1T|0 \\ T & \rightarrow & 0S|1U|1 \\ U & \rightarrow & 1T|0W \\ W & \rightarrow & 0W|1W|\varepsilon \end{array}$$