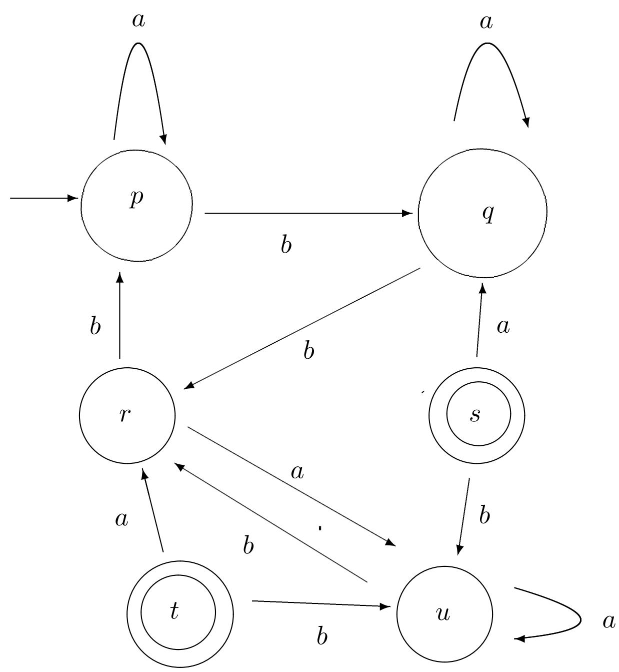
CS 420 Spring 2019
Homework 8

Due: April 17

1. Let M_1 be the Turing machine whose description is given in Example 3.9. Give the sequence of configurations that M_1 enters when started on the following input strings.
   (a) 011#011.
   (b) 110#11.
   (c) 0#0#.
2. Exercise 3.7.
3. (a) Give an implementation-level description of a one-tape Turing machine that decides the language
   $$\{w \in \{a,b,c\}^* | n_a(w) = n_b(w) \text{ and } n_a(w) > n_c(w)\}$$
   (b) Give a more efficient multi-tape Turing machine to decide the language from Part (a).
4. Problem 3.9.
5. Problem 3.11.
6. In our definition of Turing machine, if the machine tries to move left from the first tape cell, then it stays put. An alternative defintion would be that if the Turing machine tries to move left from its first cell, then it halts and rejects. (So in the alternative definition, there are two ways to reject - going to the reject state and trying to move left from the first cell.) Show how to transform a Turing machine$M$ of the type we defined in class into a Turing machine M' using this alternative defintion in such a way that M and M' recognize the same language.
7. Problem 3.15 d, e.
8. Problem 3.16 b, d.
9. Apply the method from class that decides E_{DFA} to the following DFA and answer the questions below.



(a) List the states you mark in the order they get marked.

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(b) Does the DFA belong to E_{DFA}? ________

(c) How does your answer to (b) follow from your answer to (a)?

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10. Apply the method from class that decides E_{CFG} to the following CFG and answer the questions below.

$$\begin{aligned}
S &\rightarrow aTbU|aSTb \\
T &\rightarrow YaU|bT \\
U &\rightarrow aYbY|VW \\
V &\rightarrow aV|bW \\
W &\rightarrow aW|bV \\
X &\rightarrow bX|\varepsilon \\
Y &\rightarrow aY|a|TU
\end{aligned}$$

(a) List the terminals and variables you mark in the order they get marked. (List each terminal and variable only the first time you mark it. There is more than one possible order.)

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(b) Does the CFG belong to E_{CFG}? ________________

(c) How does your answer to (b) follow from your answer to (a)?

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11. The language EQ_{REX} is defined as $\{\langle R, S\rangle | R, S$ are a regular expressions and $L(R) = L(S)\}$. Prove that EQ_{REX} is decidable.

12. Let $ALL_{NFA} = \{\langle A\rangle | A$ is an NFA and $L(A) = \Sigma^*\}$. Show that ALL_{NFA} is decidable.