CS 420 Spring 2019
Homework 9

Due: April 24

Note: Numbers in square braces refer to the numbering in the second edition of the text.

1. Let $F = \{\langle A\rangle | A$ is a DFA with input alphabet $\{0, 1\}$ and some string in $L(A)$ contains exactly three 1's$\}$. Prove that F is decidable.

2. Let $K = \{\langle A\rangle | A$ is a DFA with alphabet $\{a, b\}$ and $L(A)$ does not contain any string with exactly one more a than $b\}$ Show that K is decidable. (The solution to Problem 4.25 [4.23] is useful here.)

3. Let $L = \{\langle P\rangle | P$ is a PDA with input alphabet $\{0, 1\}$ and no string in $L(P)$ contains exactly three 1's$\}$. Prove that L is decidable.

4. Show that the set that consists of all finite sequences of natural numbers is countably infinite. In other words show that

   $$\{s| \text{ for some } n \geq 0 \text{ and natural numbers } a_1, \ldots, a_n,\ s = \langle a_1, \ldots, a_n\rangle\}$$

   is a countably infinite set.

5. Let A be a countably infinite set, B be a set, and $f : A \to B$ be onto. Prove that B is countable.

6. An infinite sequence of natural numbers $a(1)a(2)a(3)\cdots$ is called *strictly increasing* if $a(1) < a(2) < a(3) < \cdots$. Let B be the set of all strictly increasing sequences of natural numbers. Use diagonalization to prove that B is uncountable.

7. Let C be the set of infinite binary sequences $a(1)a(2)a(3)\cdots$ such that $a(1) = a(3) = a(5) = \cdots = 0$. In other words a sequence in C can have either 0 or 1 in the even positions, but has to have 0 in the odd positions.Prove that C is uncountable.