CS 420
Second Test Practice Questions

Below are some practice questions for the second test. There will be questions on the actual test that do not resemble any of these questions. Other questions on the test may ask about similar material but be phrased in a different way. Still, doing all of these practice questions will help you prepare for the test.

1. Let M_2 be the Turing machine of Figure 3.8 on page 172 of the text. For each of the following configurations of M_2, show the configuration that the given configuration yields, i.e., the next configuration after the given one. (Do not try to trace the entire computation starting from the given configuration. You are only being asked for the next configuration.)

   (a) $\sqcup q_2 00$

        Next configuration: _____

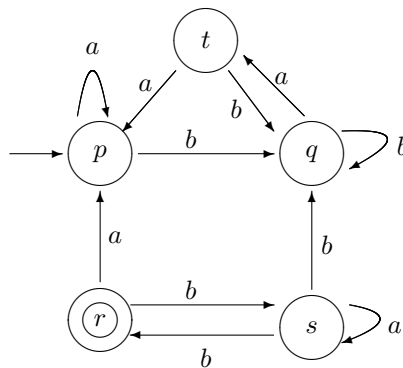   (b) $\sqcup x00q_3\sqcup$

        Next configuration: _____

   (c) $\sqcup xq_3x00$

        Next configuration: _____

   (d) $q_500\sqcup$

        Next configuration: _____

2. Apply the method from class that decides E_{DFA} to the following DFA and answer the questions below.

   (a) List the states you mark in the order they get marked.

        _____

   (b) Does the DFA belong to E_{DFA}? _____

(c) How does your answer to (b) follow from your answer to (a)?

_____

3. Apply the method from class that decides E_{CFG} to the following CFG and answer the questions below.

$$
\begin{aligned}
S &\rightarrow AB \\
A &\rightarrow Aa | BaB \\
B &\rightarrow bB | aXZ \\
X &\rightarrow \varepsilon | aX | Z \\
Z &\rightarrow ZaB | Ya \\
Y &\rightarrow aY | a
\end{aligned}
$$

(a) List the terminals and variables you mark in the order they get marked. (List each terminal and variable only the first time you mark it. There is more than one possible order.)

_____

(b) Does the CFG belong to E_{CFG}? _____

(c) How does your answer to (b) follow from your answer to (a)?

_____

4. Let $A = \{\langle M \rangle | M$ is a DFA and M accepts at least one string of odd length$\}$. Prove that A is decidable.

5. Let A be a decidable language and let B be defined by $B = \{x|$ for some string w, $xw \in A\}$. (In other words, B consists of all strings that are prefixes of strings in A.) Show that B is Turing recognizable.

6. Suppose that A and B are both countably infinite sets. Show that $A \cup B$ is also countably infinite.

7. Use diagonalization to show that $\{f : \mathcal{N} \rightarrow \mathcal{N} |$ for all n, $f(n)$ is a prime number$\}$ is uncountable.

8. Is it possible to m-reduce A_{TM} to EQ_{CFG}? Explain your answer. (You may use results in the book, on the homework, and in the homework solutions without reproving them.)

9. Let $B = \{\langle M \rangle | M$ is a Turing machine and $L(M) = (01)^*\}$. Suppose that you want to show that $A_{TM} \leq_m B$ using a reduction f that maps $\langle M, w \rangle$ to $\langle M_1 \rangle$.

(a) Fill in the blanks in the following two statements in a way that states what you have to do to make the reduction work. (In both cases you will be writing down something about the behavior of the Turing machine M_1.)

- If M accepts w, then

  _____.

- If M does not accept w, then

  _____.

(b) Give the definition of the desired Turing machine M_1, given M and w.

(c) What conclusion about B follows from the fact that $A_{TM} \leq_m B$. (You can get credit for this part without doing the previous two parts.)