CS 420 Spring 2019
Second Test Solutions

Name: ____

Put all your answers on the test itself. Be sure to put your name above.

1. Let M_2 be the Turing machine of Figure 3.8 on page 172 of the text. For each of the following configurations of M_2, show the configuration that the given configuration yields, i.e., the next configuration after the given one. (Do not try to trace the entire computation starting from the given configuration. You are only being asked for the next configuration.)

   (a) $q_1 000$
   Next configuration: $\underline{\sqcup q_2 00}$

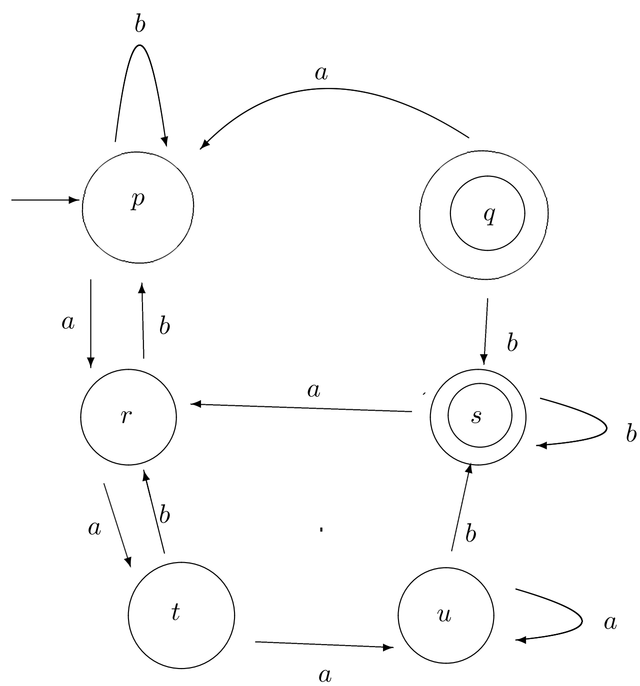
   (b) $q_5 0x0$
   Next configuration: $\underline{q_5 0x0}$

   (c) $q_5 \sqcup x0$
   Next configuration: $\underline{\sqcup q_2 x0}$

   (d) $\sqcup 0xq_3$
   Next configuration: $\underline{\sqcup 0 q_5 x}$

[24 points]

2. Apply the method from class that decides E_{DFA} to the following DFA and answer the questions below.



(a) List the states you mark in the order they get marked.
 <u>p, r, t, u, s</u>

(b) Does the DFA belong to E_{DFA}? <u>No</u>

(c) How does your answer to (b) follow from your answer to (a)?
 <u>An accept state (s) is marked.</u>

[22 points]

3. Is EQ_{TM} m-reducible to E_{TM}?

   Yes ________
   No X

   Explain your answer below. (You may use results proven in the book, in class, on the homework, and in the homework solutions without reproving them.)

   Solution: Suppose that $EQ_{TM} \leq_m E_{TM}$. Then, $\overline{EQ_{TM}} \leq_m \overline{E_{TM}}$. In class, we showed that $\overline{E_{TM}}$ is Turing recognizable, so by Theorem 5.28, $\overline{EQ_{TM}}$ is Turing recognizable. This contradicts Theorem 5.30.

   [16 points]

4. Let $DECIDER_{TM} = \{\langle M \rangle | M$ is Turing machine and M is a decider$\}$. Suppose that you want to show that $A_{TM} \leq_m DECIDER_{TM}$ using a reduction f that maps $\langle M, w \rangle$ to $\langle M_1 \rangle$.

   (a) Fill in the blanks in the following three statements in a way that states what you have to do to make the reduction work. (In all cases you will be writing down something about the behavior of the Turing machine M_1. Make your answers as general as possible.)

   - If M accepts w, then
     M_1 is a decider.
   - If M rejects w, then
     M_1 is not a decider.
   - If M loops on w, then
     M_1 is not a decider.

   (b) Give the definition of the desired Turing machine M_1, given M and w.

   M_1 = "On input x

   1. Run M on w.
   2. If M accepts, *accept*.
      If M rejects, *loop*."

[18 points]

5. Let Σ be an alphabet. Prove that the set of co-Turing recognizable languages over Σ is countably infinite.
[You are not being asked to show that each individual co-Turing recognizable language is countable. Instead, you are supposed to show that in total, the set of all possible co-Turing recognizable languages is countably infinite.]

Solution: In class, we showed that the set of all Turing-recognizable languages over Σ is countably infinite. Let $A_0, A_1, A_2, \ldots$ be a listing of these languages. Then, $\overline{A_0}, \overline{A_1}, \overline{A_2}, \ldots$ is a listing of all the co-Turing recognizable languages over Σ, so the set of co-Turing recognizable languages over Σ is countably infinite.

[10 points]

6. Let $B = \{\langle M_1, M_2\rangle | M_1, M_2$ are LBAs and $L(M_1) \nsubseteq L(M_2)\}$. Prove that B is Turing recognizable.

   Solution: B is recognized by the Turing machine N where

   $N=$ "On input $\langle M_1, M_2\rangle$ where M_1 and M_2 are LBAs

   1. Let $\Sigma^* = \{s_1, s_2, \ldots\}$
   2. For $i = 1, 2, 3, 4, \ldots$
      3. Run the Turing machine L that decides A_{LBA} on $\langle M_1, s_i\rangle$ and $\langle M_2, s_i\rangle$.
      4. If T accepts $\langle M_1, s_i\rangle$ and rejects $\langle M_2, s_i\rangle$, *accept*, else next i."

   [10 points]