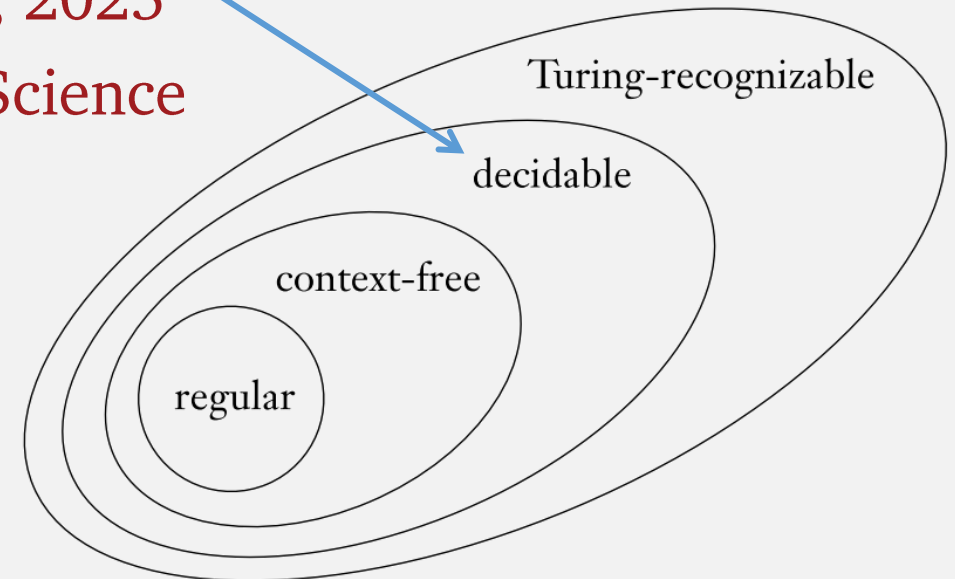


CS 420 / CS 620

# Decidability

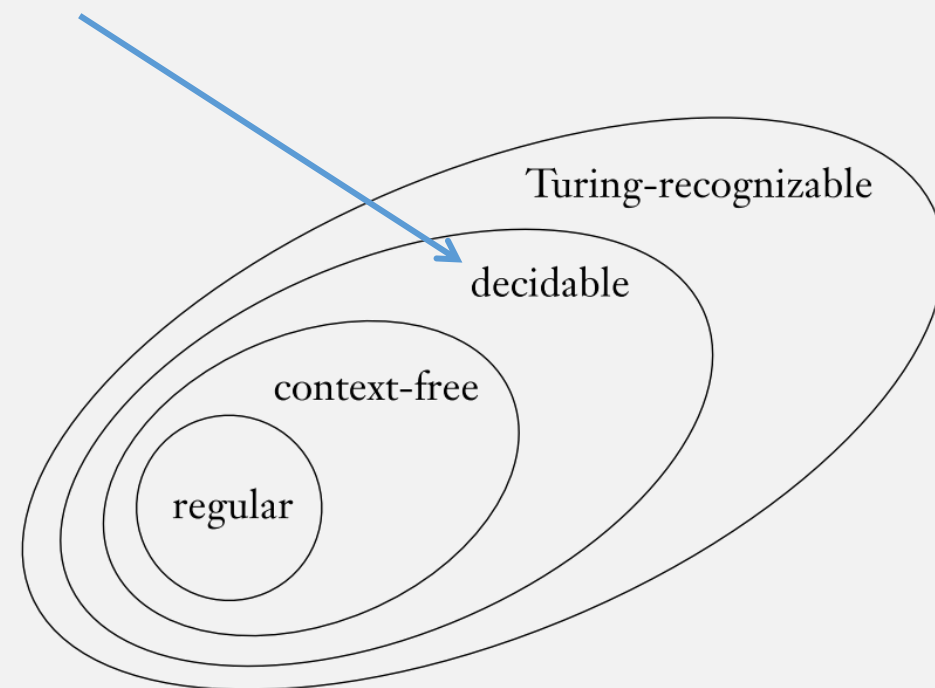
Wednesday, November 5, 2025

UMass Boston Computer Science



# Announcements

- HW 9
  - Out: Mon 11/3 12pm (noon)
  - Due: Mon 11/10 12pm (noon)



# *Previously:* Turing Machines and Algorithms

- **Turing Machines** can express more “computation” (than other prev machines)
  - Analogy: a TM models a (Python, Java) program (function)!

- 2 classes of Turing Machines

- **Recognizers:** may loop forever

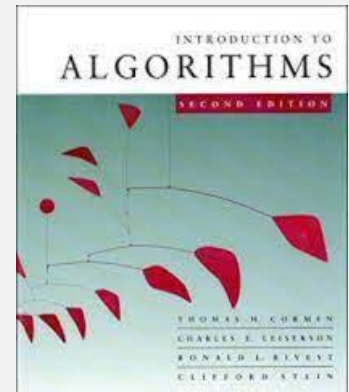
- **Deciders:** always halt

Today

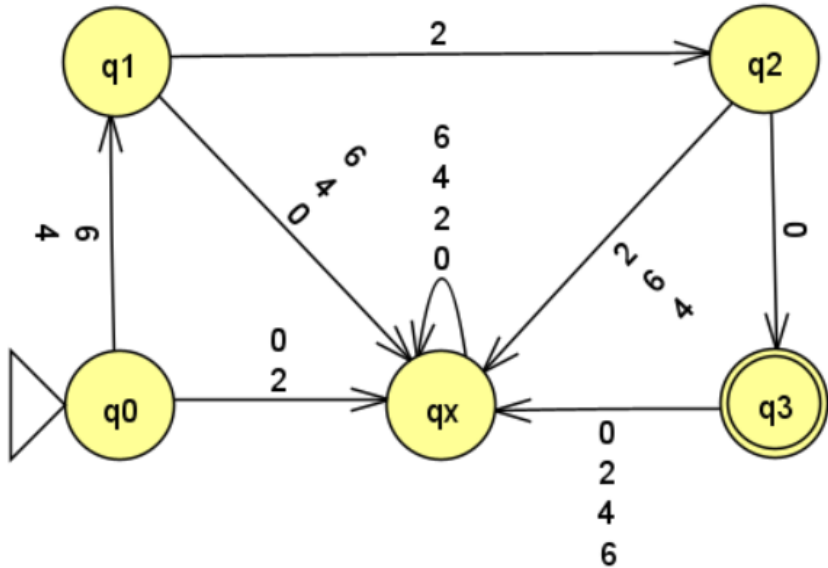


- **Deciders = Algorithms**

- I.e., an **algorithm** is a program that (for any input) always **halts**



# Flashback: HW 1, Problem 2



1. Come up with 2 strings that are accepted by the DFA. These strings are said to be "in the language" recognized by the DFA.
2. Come up with 2 strings that are not accepted (rejected) by the DFA. These strings are "not in the language" recognized by the DFA.

Your task:  
"compute" how a DFA computes

This is computation about computation: whether DFA  $B$ 's computation with input  $w$  **accepts!**

Figuring out this HW problem (about a DFA's computation) ... is itself (meta) computation!

language  
What "kind" of computation is it?

Could you write a program (function) to compute this algorithm?

A function: **DFAaccepts**( $B, w$ ) "returns" TRUE if DFA  $B=(Q, \Sigma, \delta_B, q_0, F)$  accepts string  $w$

- 1) Define "current" state  $q_{\text{current}} = \text{start state } q_0$
- 2) For each input char  $a_i \dots$  in  $w$ 
  - a) Define  $q_{\text{next}} = \delta_B(q_{\text{current}}, a_i)$  "get  $\delta_B$ "
  - b) Set  $q_{\text{current}} = q_{\text{next}}$
- 3) Return TRUE if  $q_{\text{current}}$  is in set  $F$

# The language of **DFAaccepts**

$$A_{\text{DFA}} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w \}$$

Def: a **language** is a set of strings

How is this a set of strings???

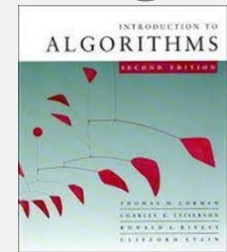
A function: **DFAaccepts**( $B, w$ ) “returns”  
TRUE if DFA  $B$  accepts string  $w$

# Interlude: Encoding Things into Strings

Definition: A language's elements / (Turing) machine's input is always a **string**

Problem: We sometimes want: TM's (program's) input to be "something else" ...

- **set, graph, DFA, ...?**



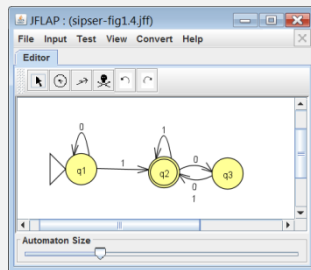
Solution: allow **encoding** "other kinds of input" as a string

Notation:  $\langle \text{SOMETHING} \rangle$  = string **encoding** for **SOMETHING**

- A tuple combines multiple encodings, e.g.,  $\langle B, w \rangle$  (from prev slide)

Details don't matter! (In this class) Can assume some encoding is always possible

Example: Possible string encoding for a DFA?



```
<automaton>
<!--The list of states.-->
<state name="q1"><initial/></state>
<state name="q2"><final/></state>
<state name="q3"></state>
<!--The list of transitions.-->
<transition>
<from>0</from>
<to>0</to>
<read>0</read>
</transition>
<transition>
<from>1</from>
```

Or:  
 $(Q, \Sigma, \delta, q_0, F)$   
(written as string)

# Interlude: High-Level TMs and Encodings

A high-level TM description, when it uses encoded input:

1. Needs to say the **type** of its input

- E.g., graph, **DFA**, etc.

$M =$  “On input  $\langle B, w \rangle$ , where  $B$  is a **DFA** and  $w$  is a string:

2. Doesn't need to say how input string is encoded

- Assume 1: input is a valid encoding

- Invalid encodings implicitly rejected

- Assume 2: TM knows how to parse and **extract parts of input**

e.g.,

Definition of  
TM  $M$  can assume:  $B = (Q, \Sigma, \delta, q_0, F)$

Details don't matter! (In this class) Can  
assume some encoding is always possible

Implicit “getters”

# DFAaccepts as a TM recognizing $A_{\text{DFA}}$

Remember:  
TM ~ program (function)  
Creating TM ~ programming

$$A_{\text{DFA}} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w \}$$

*Previously*

A function: `DFAaccepts(B,w)` “returns”  
TRUE if DFA  $B$  accepts string  $w$

- 1) Define “current” state  $q_{\text{current}} = \text{start state } q_0$
- 2) For each input char  $a_i \dots$  in  $w$ 
  - a) Define  $q_{\text{next}} = \delta(q_{\text{current}}, a_i)$
  - b) Set  $q_{\text{current}} = q_{\text{next}}$
- 3) Return TRUE if  $q_{\text{current}}$  is accept state in  $F$



TM  $M_{\text{DFA}} =$

“On inp Definition of  $A_{\text{DFA}}$  s a DFA and  $w$  is a string:

- TM  $M$  can assume:  $B = (Q, \Sigma, \delta, q_0, F)$  Implicit “getters”
- 1) Define “current” state  $q_{\text{current}} = \text{start state } q_0$
  - 2) For each input char  $a_i \dots$  in  $w$ 
    - a) Define  $q_{\text{next}} = \delta(q_{\text{current}}, a_i)$
    - b) Set  $q_{\text{current}} = q_{\text{next}}$
  - 3) **Accept** if  $q_{\text{current}}$  is an accept state in  $F$



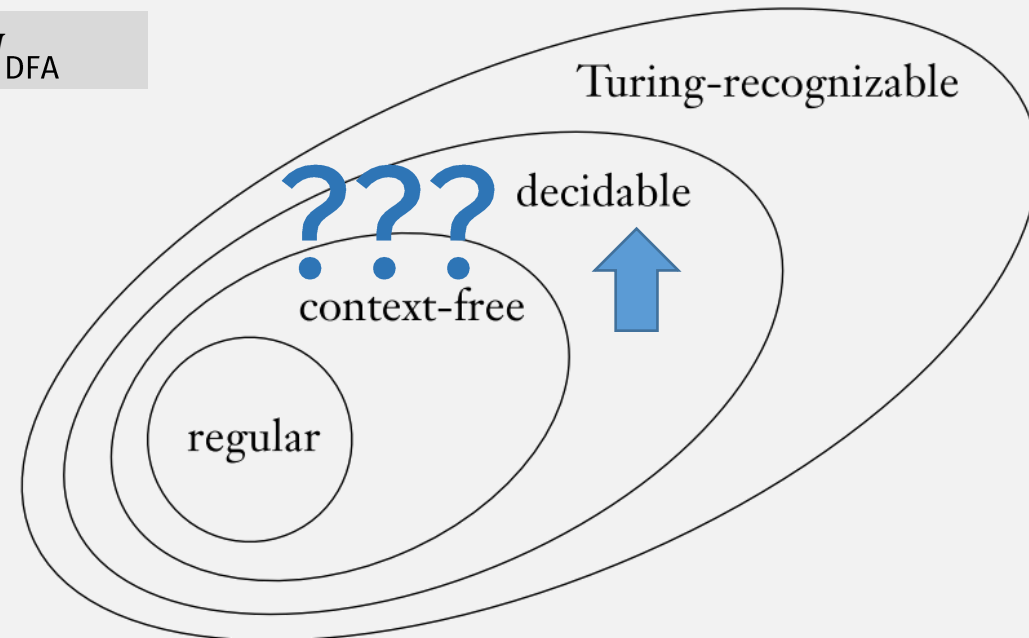
# The language of **DFAaccepts**

language  
What “kind” of computation is it?

$$A_{\text{DFA}} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w \}$$

- $A_{\text{DFA}}$  has a Turing machine  $M_{\text{DFA}}$
- Is the TM a **decider** or **recognizer**?
  - I.e., is it an **algorithm**?
- To show it's an algo, need to prove:

$A_{\text{DFA}}$  is a decidable language



How to prove that a language is decidable?

# How to prove that a language is decidable?

## Statements

1. If a **decider** decides a lang  $L$ , then  $L$  is a **decidable** lang
2. Define **decider**  $M =$  On input  $w \dots$ ,  **$M$  decides  $L$**
3.  $L$  is a **decidable** language

Key  
step

## Justifications

1. Definition of **decidable** langs
2. See  $M$  TM def, and **Examples Table**
3. By statements #1 and #2

# How to Design Deciders

- A **Decider** is a TM ...
  - See previous slides on how to:
    - write a **high-level TM description**
    - Express **encoded** input strings
  - E.g.,  $M = \text{On input } \langle B, w \rangle$ , where  $B$  is a DFA and  $w$  is a string: ...
- A **Decider** is a TM ... that must always **halt**
  - Can only: **accept** or **reject**
  - Cannot: go into an infinite loop
- So a **Decider** definition must include: an extra **termination argument:**
  - Explains how every step in the TM halts
  - (Pay special attention to loops)
- Remember our analogy: TMs ~ Programs ... so Creating a TM ~ Programming
  - To design a TM, think of how to write a program (function) that does what you want

# Thm: $A_{\text{DFA}}$ is a decidable language

$$A_{\text{DFA}} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w \}$$

Key  
step

Decider for  $A_{\text{DFA}}$  :

Decider input must match (encodings of) strings in the language!

$M =$  “On input  $\langle B, w \rangle$ , where  $B$  is a DFA and  $w$  is a string:

1. Simulate  $B$  on input  $w$ . “Calling” the DFA (with an input argument)
2. If the simulation ends in an accept state, *accept*. If it ends in a nonaccepting state, *reject*.”

Where “Simulate” =

- Define “current” state  $q_{\text{current}}$  = start state  $q_0$
- For each input char  $x$  in  $w$  ...

- Define  $q_{\text{next}} = \delta(q_{\text{current}}, x)$
- Set  $q_{\text{current}} = q_{\text{next}}$

(meta) Compute how the DFA would compute (with input  $w$ )

Remember:

**TM ~ program**

**Creating TM ~ programming**

# Thm: $A_{\text{DFA}}$ is a decidable language

$$A_{\text{DFA}} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w \}$$

Decider for  $A_{\text{DFA}}$  :

NOTE: A TM must declare “function” parameters and types ... (don't forget it)

$M =$  Undeclared parameters can't be used! (This TM is now invalid because  $B, w$  are undefined!)

1. Simulate  $B$  on input  $w$ . ← ... which can be used (properly!) in the TM description
2. If the simulation ends in an accept state, *accept*. If it ends in a nonaccepting state, *reject*.”

# Thm: $A_{\text{DFA}}$ is a decidable language

$$A_{\text{DFA}} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w \}$$

Decider for  $A_{\text{DFA}}$  :

$M =$  “On input  $\langle B, w \rangle$ , where  $B$  is a DFA and  $w$  is a string:

1. Simulate  $B$  on input  $w$ .
2. If the simulation ends in an accept state, *accept*. If it ends in a nonaccepting state, *reject*.”

Where “Simulate” =

- Define “current” state  $q_{\text{current}} =$  start state  $q_0$
- For each input char  $x$  in  $w \dots$ 
  - Define  $q_{\text{next}} = \delta(q_{\text{current}}, x)$
  - Set  $q_{\text{current}} = q_{\text{next}}$

Termination Argument: Step #1 always halts because: the simulation input is always finite, so the loop has finite iterations and always halts

Deciders must have a **termination argument**:  
Explains how every step in the TM halts (we typically only care about loops)

Thm:  $A_{\text{DFA}}$  is a decidable language

$$A_{\text{DFA}} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w \}$$

Decider for  $A_{\text{DFA}}$  :

$M =$  “On input  $\langle B, w \rangle$ , where  $B$  is a DFA and  $w$  is a string:

1. Simulate  $B$  on input  $w$ .
2. If the simulation ends in an accept state, *accept*. If it ends in a nonaccepting state, *reject*.”

Termination Argument: Step #2 always halts because:  
**determining *accept* requires checking finite number of accept states**

Deciders must have a **termination argument**:  
Explains how every step in the TM halts (we typically only care about loops)



# Thm: $A_{DFA}$ is a decidable language

$$A_{DFA} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w \}$$

Decider for  $A_{DFA}$  :

$M =$  “On input  $\langle B, w \rangle$ , where  $B$  is a DFA and  $w$  is a string:

1. Simulate  $B$  on input  $w$ .
2. If the simulation ends in an accept state, *accept*. If it ends in a nonaccepting state, *reject*.”

(New for TMs) column(s) for “called” machines

“Actual” behavior

“Expected” behavior

Example Str	$B$ on input $w$ ?	$M$ ?	In $A_{DFA}$ lang?
$\langle D_1, w_1 \rangle$	Accept	Accept	Yes
$\langle D_2, w_2 \rangle$	Reject	Reject	No

Columns must match!

Let:  
 -  $D_1 =$  DFA, accepts  $w_1$   
 -  $D_2 =$  DFA, rejects  $w_2$

(especially important when machine could loop)

# Thm: $A_{\text{NFA}}$ is a decidable language

$$A_{\text{NFA}} = \{ \langle B, w \rangle \mid B \text{ is an NFA that accepts input string } w \}$$

Decider for  $A_{\text{NFA}}$  :

Decider input must match (encodings of) strings in the language!

$N =$  “On input  $\langle B, w \rangle$ , where  $B$  is an NFA and  $w$  is a string:

1. Convert NFA  $B$  to an equivalent DFA  $C$ , using the procedure for NFA→DFA ???
2. Run TM  $M$  on input  $\langle C, w \rangle$ .
3. If  $M$  accepts, *accept*; otherwise, *reject*.”

## Flashback: NFA→DFA

Have:  $N = (Q, \Sigma, \delta, q_0, F)$

Want to: construct a DFA  $M = (Q', \Sigma, \delta', q_0', F')$

1.  $Q' = \mathcal{P}(Q)$ .

This conversion is computation!

2. For  $R \in Q'$  and  $a \in \Sigma$ ,

$$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$$

So it can be computed by a  
(**decider?**) Turing Machine

3.  $q_0' = \{q_0\}$

4.  $F' = \{R \in Q' \mid R \text{ contains an accept state of } N\}$

# Turing Machine **NFA→DFA**

New TM Variation!  
Doesn't accept or reject,  
Just writes "output" to tape

**TM NFA→DFA** = On input  $\langle N \rangle$ , where  $N$  is an NFA and  $N = (Q, \Sigma, \delta, q_0, F)$

1. Write to the tape: DFA  $M = (Q', \Sigma, \delta', q_0', F')$

Where:  $Q' = \mathcal{P}(Q)$ .

For  $R \in Q'$  and  $a \in \Sigma$ ,

$$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$$

$$q_0' = \{q_0\}$$

$$F' = \{R \in Q' \mid R \text{ contains an accept state}\}$$

Why is this guaranteed to halt?

Because a DFA description has only finite parts (finite states, finite transitions, etc)

So any loop iteration over them is finite

# Thm: $A_{\text{NFA}}$ is a decidable language

$$A_{\text{NFA}} = \{ \langle B, w \rangle \mid B \text{ is an NFA that accepts input string } w \}$$

Decider for  $A_{\text{NFA}}$  :

Remember:  
TM ~ program  
Creating TM ~ programming  
**Previous theorems ~ library**

“Calling”  
another TM.  
Must give  
correct arg type!

$N =$  “On input  $\langle B, w \rangle$ , where  $B$  is an NFA and  $w$  is a string:

1. Convert **NFA  $B$**  to an equivalent **DFA  $C$** , using the procedure  
**NFA→DFA**
2. Run TM  $M$  on input  $\langle C, w \rangle$ . ( $M$  is the  $A_{\text{DFA}}$  decider from prev slide)
3. If  $M$  accepts, *accept*; otherwise, *reject*.”

New capability:  
TM can **check tape**  
of another TM  
after calling it

Termination argument: **This is a decider (i.e., it always halts) because:**

- **Step 1** always halts bc: **NFA→DFA is decider** (finite number of NFA states)
- **Step 2** always halts because:  **$M$  is a decider** (prev  $A_{\text{DFA}}$  thm)

# How to Design Deciders, Part 2

Hint:

- Previous theorems / constructions are a “library” of reusable TMs
- When creating a TM, use this “library” to help you!
  - Just like libraries are useful when programming!
- E.g., “Library” for DFAs:
  - $NFA \rightarrow DFA$ ,  $RegExpr \rightarrow NFA$
  - $UNION_{DFA}$ ,  $STAR_{PDA}$ ,  $ENC$ ,  $reverse$
  - Deciders for:  $A_{DFA}$ ,  $A_{NFA}$ ,  $A_{REX}$ , ...

Thm:  $A_{\text{REX}}$  is a decidable language

$$A_{\text{REX}} = \{ \langle R, w \rangle \mid R \text{ is a regular expression that generates string } w \}$$

Decider:

NOTE: A TM must declare “function” parameters and types ... (don't forget it)

$P =$  “On input  $\langle R, w \rangle$ , where  $R$  is a regular expression and  $w$  is a string:

1. Convert regular expression  $R$  to an equivalent NFA  $A$  by using the procedure **RegExpr $\rightarrow$ NFA**

... which can be used (properly!) in the TM description

Remember:  
TMs ~ programs  
Creating TM ~ programming  
Previous theorems ~ library

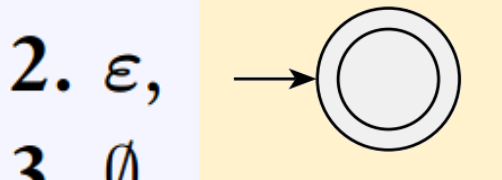
# RegExpr $\rightarrow$ NFA

... so guaranteed to always reach base case(s)

Does this conversion always halt, and why?

$R$  is a *regular expression* if  $R$  is

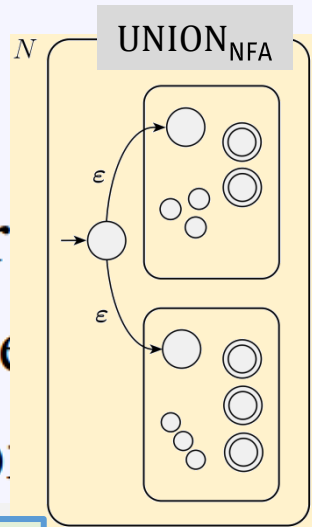
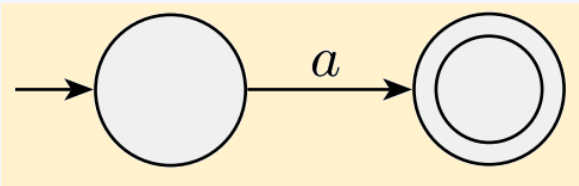
1.  $a$  for some  $a$  in the alphabet  $\Sigma$ ,



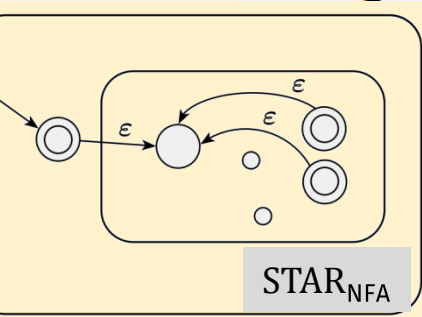
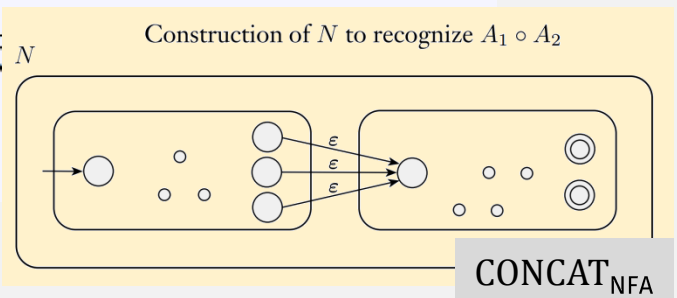
4.  $(R_1 \cup R_2)$ , where  $R_1$  and  $R_2$  are regular expressions

5.  $(R_1 \circ R_2)$ , where  $R_1$  and  $R_2$  are regular expressions

6.  $(R_1^*)$ , where  $R_1$  is a regular expression



$$\text{RegExpr} \rightarrow \text{NFA}(R_1 \cup R_2) = \text{UNION}_{\text{NFA}}(\text{RegExpr} \rightarrow \text{NFA}(R_1), \text{RegExpr} \rightarrow \text{NFA}(R_2))$$



Yes, because recursive call only happens on "smaller" regular expressions ...



# Thm: $A_{\text{REX}}$ is a decidable language

$$A_{\text{REX}} = \{ \langle R, w \rangle \mid R \text{ is a regular expression that generates string } w \}$$

Decider:

$P =$  “On input  $\langle R, w \rangle$ , where  $R$  is a regular expression and  $w$  is a string:

1. Convert regular expression  $R$  to an equivalent NFA  $A$  by using the procedure **RegExpr $\rightarrow$ NFA**
2. Run TM  $N$  on input  $\langle A, w \rangle$ . (from prev slide)
3. If  $N$  accepts, *accept*; if  $N$  rejects, *reject*.”

When “calling” another TM, must give proper arguments!

Termination Argument: This is a decider because:

- Step 1: always halts because: converting a reg expr to NFA is done recursively, where the reg expr gets smaller at each step, eventually reaching the base case
- Step 2: always halts because:  $N$  is a decider

# Decidable Languages About DFAs

Remember:  
TMs ~ programs  
Creating TM ~ programming  
Previous theorems ~ library

- $A_{\text{DFA}} = \{\langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w\}$ 
  - Decider TM: implements  $B$  DFA's extended  $\delta$  fn algorithm
- $A_{\text{NFA}} = \{\langle B, w \rangle \mid B \text{ is an NFA that accepts input string } w\}$ 
  - Decider TM: uses **NFA  $\rightarrow$  DFA** algorithm +  $A_{\text{DFA}}$  decider
- $A_{\text{REGEX}} = \{\langle R, w \rangle \mid R \text{ is a regular expression that generates string } w\}$ 
  - Decider TM: uses **RegExpr  $\rightarrow$  NFA** algorithm +  $A_{\text{NFA}}$  decider

# Flashback: Why Study Algorithms About Computing

To predict what programs will do  
(without running them!)

Not possible for all programs! But ...

```
function check(n)
{ // check if the number n is a prime
  var factor; // if the checked number is not a prime, this is its first factor
  var c;
  factor = 0;
  // try to divide the checked number by all numbers till its square root
  for (c=2; (c <= Math.sqrt(n)); c++)
  {
    if (n%c == 0) // is n divisible by c?
      { factor = c; break }
  }
  return (factor);
} // end of check function

function communicate()
{ // communicate with the user
  var i; // i is the checked number
  var factor; // if the checked number is not a prime, this is its first factor
  i = document.getElementById("number").value; // get the checked number
  // is it a valid input?
  if ((isNaN(i)) || (i <= 0) || (Math.floor(i) != i))
  { alert ("The checked object should be a whole positive number"); }
  else
  {
    factor = check (i);
    if (factor == 0)
      { alert (i + " is a prime"); }
    else
      { alert (i + " is not a prime, " + i + "=" + factor + "X" + i/factor) }
  }
} // end of communicate function
```

**RANSOMWARE ATTACK** 



???

# Predicting What Some Programs Will Do ...

What if we: look at simpler computation models  
... like DFAs and regular languages!

# Thm: $E_{\text{DFA}}$ is a decidable language

$$E_{\text{DFA}} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) = \emptyset \}$$

$E_{\text{DFA}}$  is a language ... of DFA descriptions,  
i.e.,  $(Q, \Sigma, \delta, q_0, F)$  ...

... where the language of each DFA ...  
must be  $\{\}$ , i.e., DFA accepts no strings

Is there a decider that  
accepts/rejects DFA descriptions ...

... by predicting something  
about the DFA's language  
(by analyzing its description)

Key idea / question we are about to study:  
Compute (predict) something about  
the runtime computation of a program,  
by analyzing only its source code?

Analogy  
DFA's description : a program's source code ::  
DFA's language : a program's runtime computation

Important: don't confuse the different languages here!

# Thm: $E_{\text{DFA}}$ is a decidable language

$$E_{\text{DFA}} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) = \emptyset \}$$

Decider:

$T =$  “On input  $\langle A \rangle$ , where  $A$  is a DFA:

1. Mark the start state of  $A$ .
2. Repeat until no new states get marked:
3. Mark any state that has a transition coming into it from any state that is already marked.
4. If no accept state is marked, *accept*; otherwise, *reject*.”

If loop marks at least 1 state on each iteration, then it eventually terminates because there are finite states; else loop terminates

~~(meta) Compute how the DFA would compute~~

Termination argument?

i.e., this is a “reachability” algorithm ...

... check if accept states are “reachable” from start state

Note: TM  $T$  is doing a new computation on DFAs! (It does not “run” the DFA!)

Instead: compute something about DFA's language (runtime computation) by analyzing its description (source code)

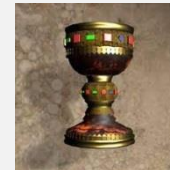
Thm:  $EQ_{\text{DFA}}$  is a decidable language

$$EQ_{\text{DFA}} = \{\langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$$

I.e., Can we compute whether  
two DFAs are “equivalent”?



Replacing “**DFA**” with “**program**” =  
A “**holy grail**” of computer science!



# Thm: $EQ_{DFA}$ is a decidable language

~~(meta) Compute how the DFA would compute i.e., run them~~

$$EQ_{DFA} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B) \}$$

I.e., Can we compute whether two DFAs are “equivalent”?

## A Naïve Attempt (assume alphabet $\{a\}$ ):

### 1. Simulate:

- $A$  with input  $a$ , and
- $B$  with input  $a$
- **Reject** if results are different, else ...

### 2. Simulate :

- $A$  with input  $aa$ , and
- $B$  with input  $aa$
- **Reject** if results are different, else ...

• ...

This might not terminate!  
(Hence it's not a decider)

Key idea

Can we compute this without running the DFAs, i.e., by only examining the DFA's “source code”?

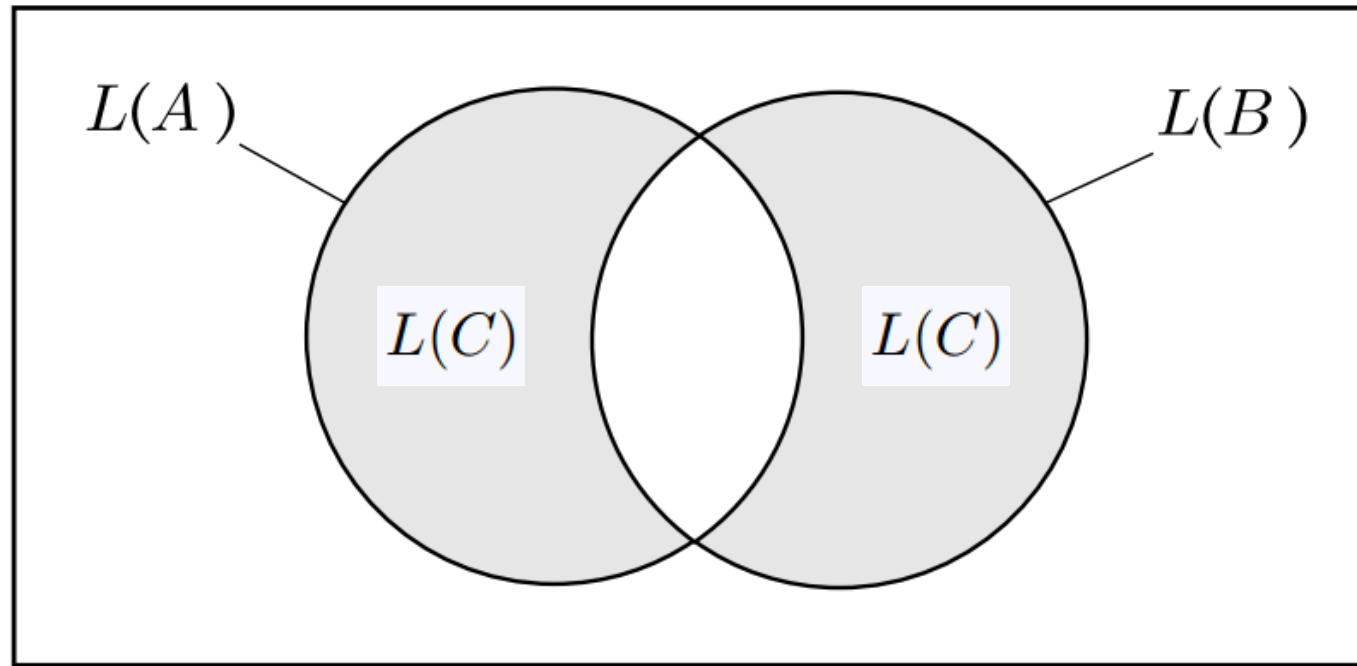


Thm:  $EQ_{\text{DFA}}$  is a decidable language

$$EQ_{\text{DFA}} = \{\langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$$

Trick: Use Symmetric Difference

# Symmetric Difference



$$L(C) = \left( L(A) \cap \overline{L(B)} \right) \cup \left( \overline{L(A)} \cap L(B) \right)$$

$$L(C) = \emptyset \text{ iff } L(A) = L(B)$$

Thm:  $EQ_{\text{DFA}}$  is a decidable language

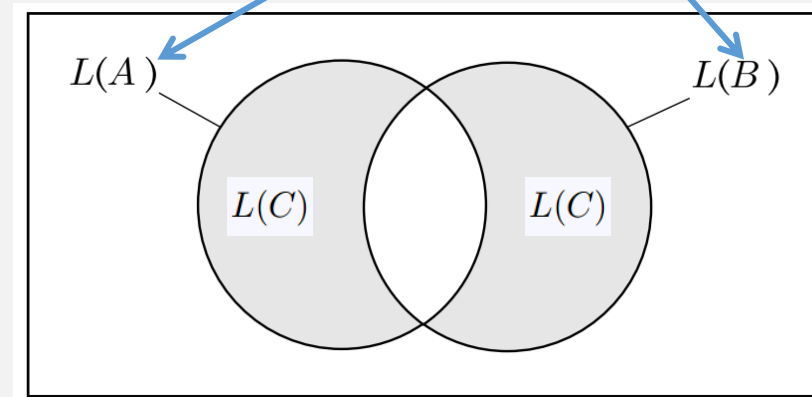
$$EQ_{\text{DFA}} = \{\langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$$

(proved in prev hws!)

NOTE, This only works because:  
regular langs closed under **negation**,  
i.e., set complement, **union** and **intersection**

Construct **decider** using 2 parts:

1. Symmetric Difference algo:  $L(C) = (L(A) \cap \overline{L(B)}) \cup (\overline{L(A)} \cap L(B))$ 
  - Construct  $C$  = Union, intersection, negation of machines  $A$  and  $B$
2. Decider  $T$  (from “library”) for:  $E_{\text{DFA}} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$ 
  - Because  $L(C) = \emptyset$  iff  $L(A) = L(B)$



Thm:  $EQ_{\text{DFA}}$  is a decidable language

$$EQ_{\text{DFA}} = \{\langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$$

TM input must use same string encoding as lang

Construct **decider** using 2 parts:

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  - Because  $L(C) = \emptyset$  iff  $L(A) = L(B)$

$F$  = “On input  $\langle A, B \rangle$ , where  $A$  and  $B$  are DFAs:

1. Construct DFA  $C$  as described.
2. Run TM  $T$  deciding  $E_{\text{DFA}}$  on input  $\langle C \rangle$ .
3. If  $T$  accepts, *accept*. If  $T$  rejects, *reject*.”

Termination argument?

# Predicting What Some Programs Will Do ...

microsoft.com/en-us/research/project/slam/

SLAM is a project for checking that software satisfies critical behavioral properties of the interfaces it uses and to aid software engineers in designing interfaces and software that ensure reliable and correct functioning. Static Driver Verifier is a tool in the Windows Driver Development Kit that uses the SLAM verification engine.

*"Things like even software verification, this has been the Holy Grail of computer science for many decades but now in some very key areas, for example, driver verification we're building tools that can do actual proof about the software and how it works in order to guarantee the reliability."* Bill Gates, April 18, 2002. [Keynote address at WinHec 2002](#)



Static Driver Verifier Research Platform README

## Overview of Static Driver Verifier Research Platform

Static Driver Verifier (SDV) is a compile-time static verification Research Platform (SDVRP) is an extension to SDV that allows

- Support additional frameworks (or APIs) and write custom
- Experiment with the model checking step.

### Model checking

From Wikipedia, the free encyclopedia

In computer science, model checking or property checking is a method for checking whether a finite-state model of a system meets a given specification (also known as correctness). This is typically

Its "language"

# Summary: Algorithms About Regular Langs

- $A_{\text{DFA}} = \{\langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w\}$ 
  - **Decider:** Simulates DFA by implementing extended  $\delta$  function

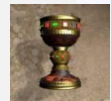
- $A_{\text{NFA}} = \{\langle B, w \rangle \mid B \text{ is an NFA that accepts input string } w\}$ 
  - **Decider:** Uses NFA  $\rightarrow$  DFA decider +  $A_{\text{DFA}}$  decider

- $A_{\text{REX}} = \{\langle R, w \rangle \mid R \text{ is a regular expression that generates string } w\}$ 
  - **Decider:** Uses RegExpr  $\rightarrow$  NFA decider +  $A_{\text{NFA}}$  decider

- $E_{\text{DFA}} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$ 
  - **Decider:** Reachability algorithm

Lang of the DFA

- $EQ_{\text{DFA}} = \{\langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$



- **Decider:** Uses complement and intersection closure construction +  $E_{\text{DFA}}$  decider

Remember:  
TMs ~ programs  
Creating TM ~ programming  
Previous theorems ~ library

*Next:* Algorithms (Decider TMs) for CFLs?

- What can we predict about CFGs or PDAs?