CS420 Homework 4
Due Tues Oct 13 at noon
Submit pdf named hw4.pdf
Submit to the CS Server in your course directory in hw4 subdirectory

## 1 A Function on Languages

Consider a function, called MAKECS, which converts an input language to a new output language such that for each string w in the input language, there is a corresponding string in the output language that starts with the characters cs and ends in the characters of w .

Formally:

MAKECS(L) = {csw | where w is a string $\in$ L}

If the alphabet for a language L is $\Sigma$, then the alphabet for MAKECS(L) is $\Sigma \cup \{c, s\}$.

Note: Though the problem is straightforwardly solvable as written, if it helps, you may assume that the alphabet for L does not contain c or s.

1. Consider the following language:

B = {w | w is a three digit number}

The alphabet for the language is $\Sigma = \{2, 4\}$. Prove that B is a regular language.

2. What is the language MAKECS(B)? (answer must be a set)

3. Prove that MAKECS(B) is a regular language

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## 2 Are Regular Languages Closed Under MAKECS?

Prove that the class of regular languages is closed under the MAKECS operation.
In other words, prove if the input language L is a regular language, then MAKECS(L) is a regular language.

Hint:

- Give the equivalent IF-THEN statement that must be proved.
- Give a proof of this IF-THEN statement.
- Make sure your answer is in the form of a "Statements and Justifications" table, as explained in lecture (and also Hopcroft Chapter 1).
- Since we don't know the input language L, if you need to give examples in your proof, you'll need to express them in terms of the unknown input languages, e.g.: "say we have a language L and string s1 $\in$ L and s2 $\in$ L, then we know that ?? $\in$ MAKECS(L) and ?? $\in$ MAKECS(L)"
- You should have at least 2 examples, which must include at least one that is in the output language and one that is not in the output language.

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3 DFAs vs NFAs

Consider the diagram from Homework 1 DFA Formal Description: Diagram is posted on the website, from problem2 in Homework 1. It has the bright yellow states.

For this problem, assume the diagram is an NFA.

In class we learned that an NFA's formal description has five components, e.g. $N = (Q, \Sigma, \delta, q_{start}, F)$, where $\delta : Q \times \Sigma\varepsilon \rightarrow P(Q)$ is the transition function mapping a state

and input symbol (or no symbol, in the case of an empty transition) to a set of states.

1. Come up with a formal NFA description of the diagram above (yes, it's the same diagram as HW1, but now you are being told it an NFA).

2. Compare your answers from this problem and Homework 1 and give two differences between the formal description of a DFA and the formal description of an NFA.