UMass Boston cs444 hw2
Parity Bits, Hamming Codes and RAID 2
Posted Tuesday, November 11, 2025
Due Tuesday, November 18, 2025 at 11:59 pm

J.H.DeBlois

Homework must be typed and converted to Portable Document Format (PDF), see `https://en.wikipedia.org/wiki/PDF`. We use the Linux servers of the Computer Science Department to collect homework.

To submit your homework, prepare one PDF file called `hw2.pdf`. The filename must be exactly `hw2.pdf`, otherwise it will not be collected. Upload the file to the `cs444` folder linked to your home directory on the CS Linux server. If you have trouble with uploading, email `operator@cs.umb.edu` for help.

The questions in this homework are based on the Hamming slides, to be presented in class Thursday, November 12th. Also refer to the wikipedia articles on Parity Bit `https://en.wikipedia.org/wiki/Parity_bit` and ECC Memory `https://en.wikipedia.org/wiki/ECC_memory`. And if you like take a look at the website `https://graphics.stanford.edu/~seander/bithacks.html`, which offers many great uses of bit operations.

1 Parity Bits

Consider one byte with 8 bits. Suppose there are 7 bits of data, so there is one bit left free in the byte. If that bit is the parity bit, then it can be used by two machines to determine whether a sent byte has developed an error during transmission.

Let the message be this byte: 00110011. Let the "high bit" (the most significant bit, all the way to the left) be the parity bit.

Let the established parity be even parity.

1.1

(5 points) In the byte of the message, is the parity bit set correctly? (yes or no)

1.2

(5 points) Show the calculation you used to figure that out:

1.3

(5 points) Switch the established parity to odd parity. Recompute the parity bit for the same 7 bits of the message. Rewrite the message byte:

2 Hamming Code

Hamming codes have a special pattern for where the parity bits are inserted. Refer to the slides for explanations.

2.1

(6 points) For Hamming(15,11), what is the code for binary 1010 1001 010? What is the code for binary 1010 1101 010?

2.2

(6 points) What is the Hamming distance between the codes above?

2.3

(6 points) For Hamming(7,4), what does the number 7 mean? For Hamming(7,4), what does the number 4 mean?

2.4

(7 points) What is the Hamming(7,4) code for binary 1111?

3 Hamming Code and RAID 2

Hamming(7,4) code adds 3 parity bits to 4 data bits to create 7-bit codes. RAID 2 puts each bit on a separate disk so there would have to be 7 disks. Since you cannot write anything smaller than a byte to a disk, you would have to work with 8 Hamming(7,4) codes in order to get 8 bits from different codes to have a byte ready to write out to each disk.

Let the data be four bits: 0101. Hamming(7,4) adds 3 parity bits, usually named P1, P2 and P4. For this example, name them p, q and r. This gives the code: pq0r101.

The established parity is even parity. You compute the parity of the 4 data bits 3 at a time in a pattern. For P1, take even parity of D1, D2 and D4. For P2, take even parity of D1, D3, D4. For P4, take even parity of D2, D3 and D4.

3.1

(8 points) Using the given data bits, compute P1:

3.2

(8 points) Using the given data bits, compute P2:

3.3

(8 points) Using the given data bits, compute P4:

3.4

(8 points) Since P1 is p, P2 is q, and P3 is r, fill in the values to create the code byte pq0r111 =

3.5

(8 points) RAID 2 writes a bit of each code onto each of 7 disks. We cannot write until we have a bytes. For the first byte to write out, read the first bit from each of the eight codes. Byte 1 to write out will be: pppppppp. Use your value of p to fill it in: byte1: pppppppp =

3.6

(8 points) The byte3 is all zeroes. Write the other 5 bytes using the variables and the data bits:

byte2:
byte3: 00000000
byte4:
byte5:
byte6:
byte7:

Then add equals signs to the ones with variables and fill in your parity values and the data bits.

3.7

(12 points) Now suppose the first RAID disk fails. Using Error Correction provided by RAID 2 redundancy, figure out the value of the P1 for each of the bits on the drive that failed.

Since we only wrote out one byte per drive, you have to figure out the 8 bits of data that got lost by using the data on the 6 other drives. Explain the steps to do this. Be specific.