Semi-Riemannian Geometry and Relativity

Informal Seminar

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Location June 4–July 26
M Th 5:00–6:30pm
Taffee Tanimoto Conference Room (S-3-180)

Description General relativity, Einstein's formulation of gravity, is written in the language of differential geometry. In this informal seminar, we will start with the basic vocabulary and grammar of the language, beginning with manifolds and metrics, and work our way up to the field equations themselves. Along the way, we will investigate special relativity, tensor calculus, and curvature. The material surrounding general relativity is rich. Other, possible topics of interest include methods from calculus of variations, complex geometry, time orientability and causality, cosmological models, and the geometry of various kinds of black holes.

Background Necessary background preparation is somewhat modest, given the myths surrounding relativity and its founders. Participants need only have a working sense of differentiation and integration in a single variable (though knowing how they work in a multivariable context is more useful), and some linear algebra. In particular, conceptual knowledge of the chain rule and, to a lesser extent, integration by parts will prove indispensable. Determinants, traces, and eigenvalues will show up often and in unexpected places.

An intuition from point set topology is also helpful but by no means necessary. I expect to build most topics from the ground up. And we'll practice reading and writing mathematical proofs as a matter of course.

Rough Outline Below is a tentative skeleton of topics.

0 Preliminaries

· Point set topology: open sets, continuous functions
· Linear algebra: bases, trace, eigenvalues, eigenvectors
· Analysis: limits, derivatives

1 Calculus on Manifolds

· Smooth manifolds, smooth mappings
· Tangent vectors, fields, the tangent bundle and their duals
· Tensor operations

2 Semi-Riemannian Manifolds

· Metric tensors and isometries
· Parallel transport, connections, and derivative operators
· Curvature: Riemannian, Ricci, sectional, and scalar
· Geodesics
· Computing Curvature: Cartan's Structure Equations

3 Semi-Riemannian Submanifolds

4 Riemannian and Lorentz Geometry
· Arc length, Riemannian distance
· Lorentz vector spaces
· Local Lorentz geometry
· Poincaré and Schwarzchild half-planes

5 Special Relativity
· Bondi calculus
· Newtonian space, time, and space-time
· Minkowski space-time and geomety
· Some relativistic effects

6 General Relativity
· Foundations, physical motivation
· The Einstein field equations
· Cosmological models
· Palatini formulation, Einstein-Schrödinger-Straus Theory

7 Geometry of a Schwarzchild Black Hole

REFERENCES While I won't follow any particular book, I will draw from a few usual suspects. (I'll try to write up lecture notes in PDF.) Texts that I think would be especially useful for this seminar are marked in **bold**. This list is not exhaustive. If you have a special interest in a topic, I might be able to dig up another title for you.

Boothby, W. *An Introduction to Differentiable Manifolds and Riemannian Geometry* (2d ed.), Academic Press: 2002.
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DARLING, R. *Differential Forms and Connections*, Cambridge Univ. Press: 1994.
GELFAND, I. *Calculus of Variations*, S. V. Fomin (trans.), Prentice-Hall, Inc.: 1963.
HAWKING S., ELLIS G.F. *Large-scale Structure of Space-time*, Cambridge Univ. Press: 1975.
d'Inverno, R. *Introducing Einstein's Relativity*, Oxford Univ. Press: 1992.
Kay, D. *Schaum's Outline of Tensor Calculus*, McGraw-Hill: 1988.
LIGHTMAN, A., ET AL. *Problem Book in Relativity and Gravitation*, Princeton Univ. Press: 1975.
O'Neill, B. *Semi-Riemannian Geometry with Applications to Relativity*, Academic Press: 1983.
POOR, W. *Differential Geometric Structures*, Dover Publications: 2007.
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TAYLOR, M. *Partial Differential Equations* I*: Basic Theory*, Springer: 1996.
— *Partial Differential Equations* III*: Nonlinear Equations*, Springer: 1996.
Wald, R. *General Relativity*, Univ. of Chicago Press: 1984.