More Practice for Predicate Logic

Important points:

- Define propositional functions in a useful and reusable manner, just like functions in a computer program.
- Make sure your formalized statement evaluates to "true" in the context of the original statement and evaluates to "false" whenever the original statement is violated.

More Practice for Predicate Logic

More Examples:

Jenny likes all movies that Peter likes (and possibly more).
$\forall x\ [\text{Movie}(x) \land \text{Likes}(\text{Peter}, x) \rightarrow \text{Likes}(\text{Jenny}, x)]$

There is exactly one UMass professor who won a Nobel prize
$\exists x[\text{UMBProf}(x) \land \text{Wins}(x, \text{NobelPrize})] \land$
$\neg\exists y,z[y \neq z \land \text{UMBProf}(y) \land \text{UMBProf}(z) \land$
$\text{Wins}(y, \text{NobelPrize}) \land \text{Wins}(z, \text{NobelPrize})]$

Rules of Inference for Quantified Statements

$\forall x\ P(x)$ / $\therefore P(c)$ if $c \in U$	Universal instantiation
$P(c)$ for an arbitrary $c \in U$ / $\therefore \forall x\ P(x)$	Universal generalization
$\exists x\ P(x)$ / $\therefore P(c)$ for some element $c \in U$	Existential instantiation
$P(c)$ for some element $c \in U$ / $\therefore \exists x\ P(x)$	Existential generalization

Rules of Inference for Quantified Statements

Example:

Every UMB student is a genius.
George is a UMB student.
Therefore, George is a genius.

U(x): "x is a UMB student."
G(x): "x is a genius."

Rules of Inference for Quantified Statements

The following steps are used in the argument:

Step 1: $\forall x\ (U(x) \rightarrow G(x))$	Hypothesis
Step 2: $U(\text{George}) \rightarrow G(\text{George})$	Univ. instantiation using Step 1
Step 3: U(George)	Hypothesis
Step 4: G(George)	Modus ponens using Steps 2 & 3

$\forall x\ P(x)$ / $\therefore P(c)$ if $c \in U$ — Universal instantiation

Proving Theorems

Direct proof:

An implication $p \rightarrow q$ can be proved by showing that if p is true, then q is also true.

Example: Give a direct proof of the theorem "If n is odd, then n^2 is odd."

Idea: Assume that the hypothesis of this implication is true (n is odd). Then use rules of inference and known theorems to show that q must also be true (n^2 is odd).

Proving Theorems

n is odd.

Then $n = 2k + 1$, where k is an integer.

Consequently, $n^2 = (2k + 1)^2$.
$= 4k^2 + 4k + 1$
$= 2(2k^2 + 2k) + 1$

Since n^2 can be written in this form, it is odd.

Proving Theorems

Indirect proof:

An implication $p \rightarrow q$ is equivalent to its **contra-positive** $\neg q \rightarrow \neg p$. Therefore, we can prove $p \rightarrow q$ by showing that whenever q is false, then p is also false.

Example: Give an indirect proof of the theorem "If 3n + 2 is odd, then n is odd."

Idea: Assume that the conclusion of this implication is false (n is even). Then use rules of inference and known theorems to show that p must also be false (3n + 2 is even).

Proving Theorems

n is even.

Then $n = 2k$, where k is an integer.

It follows that $3n + 2 = 3(2k) + 2$
$= 6k + 2$
$= 2(3k + 1)$

Therefore, 3n + 2 is even.

We have shown that the contrapositive of the implication is true, so the implication itself is also true (If 3n + 2 is odd, then n is odd).