CS 320L – Applied Discrete Mathematics – Spring 2018
Instructor: Marc Pomplun

Assignment #2

Posted on March 2 – due by March 20, 5:30pm

Note: This is a fillable PDF, and you need to put all of your answers into the answer boxes. Press CTRL + E to add formatting, and you can copy and paste symbols from this or other documents (like these: ∀ ∃ → ↔ ≤ ≥ × · ≠ ∩ ∪ ⊂ ⊆ ∈ ∉ ¬). You can do this digitally, without any paper involved, or you can print this assignment, fill in the answers with a pen, and then scan the result. The copiers in the department or in McCormack can scan and email you the scanned pages. Alternatively, there are scanning apps for all types of smart phones.

Afterwards, upload your answers to gradescope.com. If you have not signed up yet, please do so using code 9D5E5J.

Question 1: Prime Factorization

Write down the prime factorization (in ascending order) of each of the following integers (Example: $720 = 2^4 \cdot 3^2 \cdot 5$).

a) 264

b) 1000000

c) 2430

d) 117

Question 2: Euclidean Algorithm

Use the Euclidean algorithm to determine the following greatest common divisors. Write down every step in your calculation.

a) gcd(3330, 550)

b) gcd(178, 300)

c) gcd(912, 625)

Question 3: Rules of Inference

Use rules of inference to show whether the following arguments are valid or not. Formalize the hypotheses and conclusion, and use the step-by-step method we discussed in class. If an argument is not valid, provide a counterexample.

a) Whenever Hans, the Elephant, sees a mouse, he runs. Whenever Hans runs, the ground is shaking. Whenever the ground shakes, Heidi, the chipmunk, gets angry. Heidi did not get angry. Therefore, Hans did not see a mouse.

b) No UMB professors are Yankees fans. Boris is a Yankees fan. Therefore, Boris is not a UMB professor.

c) Whenever the Red Sox win, Carl celebrates. Whenever Carl celebrates, he drinks some beer. Carl drinks some beer. Therefore, the Red Sox won.

Question 4: Finding Relatively Prime Numbers

As you know, two numbers are relatively prime if they do not share any divisors greater than 1. Write a function in Java, C, C++, Python, or pseudocode that takes two integers as its input and returns a Boolean value. This value is *true* if the two inputs are relatively prime and is *false* otherwise. Write your program into the box on the next page.

Question 5: Matrices

Find a matrix M such that

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} M + \begin{bmatrix} -6 & 3 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 27 \\ 6 & 15 \end{bmatrix}$$

Hint: For each column of M you have to solve a system of linear equations. Write down all of these equations and every step of their solution.

Question 6: Proofs

Prove or disprove the following statements:

a) $2^n + 3$ is prime for all positive integers n.

b) The product of three odd integers is odd.

c) For every positive integer n, n(n + 1) is even.

Question 7: Formalization of Logical Expressions

Write down the following statements using predicate logic. Give your propositional functions intuitive names so that you do not have to explain them.

a) Andreas beats up all professors that fail him and do not fail his sister Susanne.

b) Franziska and Albert never attend the same classes.

c) There is no computer scientist who can dance and sing.

Question 8: Recursion

Give a recursive definition of each of the following sequences (n = 1, 2, 3, …):

a) $a_n = n$

b) $a_n = 2n^2$

c) $a_n = 3n!$

Question 9: Converting Back and Forth

Show every step of your computation for the following conversions:

a) Convert the decimal number 1738 into its hexadecimal expansion.

b) Convert the binary number $(1001101)_2$ into its decimal expansion.

Question 10: Some Induction

Use mathematical induction to show that the following equation is true for all natural numbers n:

$2^0 + 2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$