Three Logicians Walk into a Bar (or why logic is not so scary)

Fall 2013

Nov. 7, 2013

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- The science of formal reasoning.
- The study of the structure of arguments.
- Study of formal inference and proofs.

- Electronic circuits using Boolean logic (True/False)
- Formulation of algorithms as logical statements using sets, functions, relations...
- Computability, completeness, complexity...
- AI verification and proof systems
- Syntax and semantics of programming languages
- Many many more...

A proposition is a logical sentence, consisting of a *subject* and a *predicate*

- Today is Thursday
- All cats have four legs
- Some cats are dogs

A proposition may be true or false.

Can be viewed as a function from a statement to $\{0,1\}$ (True, False)

- A set an unordered, non-redundant bunch of elements
- Examples:

 $\{11, 13, 17, 19\}$, all cats, the letters in the alphabet, the people in this room...

• Logic defines relations between sets



- A deductive argument in which conclusion is inferred from two premises.
- 3 terms and 3 propositions major, minor and conclusion

Example:

- All men are mortal
- Ø Robert is a man
- Hence, Robert is Mortal

Syllogism – A Common Pitfall

All cats have four legs

② This chair has four legs

e Hence, this chair is a cat



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• All cats have four legs • $A \rightarrow B$

• C \rightarrow B

Cats4 legs4 leqsTha777

- Hence, this chair is a cat
 - What is the relationship between A and C?

• if it rains then there are clouds

• $A \rightarrow B$

is equivalent to...

- if it doesn't rains then there are no clouds
 - not $A \rightarrow$ not B

???

• if it rains then there are clouds

• $A \rightarrow B$

- is equivalent to...
 - if it doesn't rains then there are no clouds

• not $A \rightarrow$ not B

???

- if there are no clouds then it doesn't rain
 - not $B \rightarrow not A$

Venn Diagrams



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- Truth values (True/False) are assigned to propositions.
- A truth function is a function from a set of truth values to truth values.
- Common examples Negation ("NOT"), Conjunction ("AND"), Disjunction ("OR"), exclusive disjunction ("XOR").

Truth Tables – "NOT", "AND", "OR", "XOR"









A	В	$A \lor B$
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F







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Three Logicians are Wearing Hats





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Three Logicians are Wearing Hats



*ODDS OF FAILING TWO TIMES IN A ROW.

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Puzzle: Where is the Money?

Three doors:

- An iPod
- A million dollars
- A melting popsicle

Rules:

- The \$1M door tells the truth
- The popsicle door lies
- The iPod door can go either way

A. iPod is here



B. Popsicle behind C



C. iPod behind A



- Q 1: Where is the money? Q 2: Where is the iDad2
- Q 2: Where is the iPod?

Based on "The lady or the tiger", Times books, 1982

Nurit Haspel

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