Three Logicians Walk into a Bar  
(or why logic is not so scary)

Fall 2013

Nov. 7, 2013
What is Logic?

- The science of formal reasoning.
- The study of the structure of arguments.
- Study of formal inference and proofs.
Logic in Computer Science

- Electronic circuits using Boolean logic (True/False)
- Formulation of algorithms as logical statements using sets, functions, relations...
- Computability, completeness, complexity...
- AI – verification and proof systems
- Syntax and semantics of programming languages
- Many many more...
A proposition is a logical sentence, consisting of a *subject* and a *predicate*

- *Today is Thursday*
- *All cats have four legs*
- *Some cats are dogs*

A proposition may be true or false. Can be viewed as a function from a statement to \{0, 1\} (True, False)
A set is an unordered, non-redundant bunch of elements.

Examples: \{11, 13, 17, 19\}, all cats, the letters in the alphabet, the people in this room...

Logic defines relations between sets.

**Disjoint** (no cats have 4 legs)

\[ A \subseteq B \] (all 4 legged creatures are cats)

**Intersecting** (some cats have 4 legs)

\[ B \subseteq A \] (all cats have 4 legs)
Syllogism

A deductive argument in which conclusion is inferred from two premises.

3 terms and 3 propositions - major, minor and conclusion

Example:

1. All men are mortal
2. Robert is a man
3. Hence, Robert is Mortal
1. All cats have four legs

2. This chair has four legs

3. Hence, this chair is a cat
1. All cats have four legs
   - $A \rightarrow B$

2. This chair has four legs
   - $C \rightarrow B$

3. Hence, this chair is a cat
   - What is the relationship between $A$ and $C$?
- if it **rains** then **there are clouds**
  - $A \rightarrow B$

is equivalent to...

- if it **doesn’t rain** then **there are no clouds**
  - $\neg A \rightarrow \neg B$

???
if it rains then there are clouds
\[ A \rightarrow B \]
is equivalent to...
if it doesn’t rains then there are no clouds
\[ not\ A \rightarrow not\ B \]

if there are no clouds then it doesn’t rain
\[ not\ B \rightarrow not\ A \]
Venn Diagrams

- A
- not A
- B
- not B

- A or B
- not A and not B
- A and B
- not (A and B)

- if A then B
- if B then A
- Aiff B
- Aiff not B

- A and not B
- B and not A
- Tautology
- Contradiction

Nurit Haspel

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Truth values (True/False) are assigned to propositions. A truth function is a function from a set of truth values to truth values. Common examples – Negation ("NOT"), Conjunction ("AND"), Disjunction ("OR"), exclusive disjunction ("XOR").
Truth Tables – "NOT", "AND", "OR", "XOR"

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Three Logicians Walk into a Bar

Three Logicians Walk into a Bar...

1. A: Does everyone want beer?
2. A: I don't know.
3. B: I don't know.
4. C: Yes!
Three Logicians are Wearing Hats

I don't know... Me either... SOMEHOW I KNOW THAT I AM WHITE HAT GUY!

No Clue I'unno MY HAT IS WHITE!
Three Logicians Meet at a Bar Wearing Hats...

- HI GUYS!
- READY?
- YUP.

- 3... 2... 1...

- MY HAT IS...
- BLUE!

- DAMN!
- WE FAILED LAST WEEK TOO!
- REMEMBER?

- UH... 1 IN 16...
- HA YA, WHAT ARE THE ODDS?*

- WELL OBVIOUSLY, IT WAS A FIGURE OF SPEECH.

* ODDS OF FAILING TWO TIMES IN A ROW.
Puzzle: Where is the Money?

Three doors:
- An iPod
- A million dollars
- A melting popsicle

Rules:
- The $1M door tells the truth
- The popsicle door lies
- The iPod door can go either way

A. iPod is here
B. Popsicle behind C
C. iPod behind A

Q 1: Where is the money?
Q 2: Where is the iPod?

Based on "The lady or the tiger", Times books, 1982