Final Review

CS 220 — Applied Discrete Mathematics

May 14, 2025

UMass Boston

Final Exam Information

Wednesday, May 21
from 3:00pm to 6:00pm
in Wheatley W01-0004

- Written exam. (Bring something to write with!)
- You may bring up to 40 pages of **handwritten notes**.
  (That is, 40 pieces of **loose paper**, up to letter size.)
  No printouts, no photocopies.
- No other resources: no books, no computers, no friends,
  no cellphones/smartphones/tablets.

Final Exam Topics

The final exam is **cumulative**:

- relies on knowledge from entire semester
- emphasis on topics after midterm exam

New Modules

06 Integers
07 Proofs
08 Recursion and Induction
09 Computation
10 Counting
11 Probability
12 Graphs

06 Integers

- divides ($a \mid b$), factor, multiple
- prime, composite
- fundamental theorem of arithmetic, prime factorization
- division: dividend, divisor, quotient, remainder
- greatest common divisor, least common multiple
- relatively prime
- modulo operator
- congruence modulo m ($a \equiv b \pmod{m}$)
- representing integers, modular arithmetic
- representing negative integers
- integers in other bases, conversion between bases
- arithmetic (addition, multiplication) in other bases
- two's complement

07 Proofs

- ~~proof system, deductive system~~
- axioms, theorems, lemmas, corollaries, conjecture
- ~~rules of inference, formal proofs~~
- informal proofs

08 Recursion and Induction

- recursively-defined functions and sequences
- recursively-defined sets
- recursively-defined relations
- **induction**
- induction with non-zero base
- ~~strong induction~~
- ~~structural induction~~

09 Computation

- ~~analogy machines~~
- algorithm, decision procedure
- ~~computable functions~~
- ~~termination measure~~, ~~loop invariant~~
- ~~Big-O notation~~
- Euclid's algorithm (GCD)
- ~~extended Euclidean algorithm, multiplicative inverse~~

10 Counting

- **addition (sum rule)** — for counting *"either-or"* situations, where an outcome is produced by either one task or another
  - **subtraction** — for excluding elements
  - **inclusion-exclusion** — corrects overcount of overlapping sets
- **multiplication (product rule)** — for counting *"both-and" situations*, where an outcome is created by joining multiple task outputs
  - **quotient** — correct for overcounting by a factor
  - **permutations** — for counting *ordered lists without duplicates* of elements from some set
  - **combinations** — for counting *(unordered) sets (without duplicates)* of elements from some set
- ~~**recursive equations** — (like Pascal's formula)~~
- **enumeration** — "just count", "tree diagram", etc
- **pigeonhole principle** — for *minimum size of largest group*

11 Probability

- experiment, sample space, outcome, event
- probability of event
- complementary event
- unions of events (disjoint, non-disjoint)
- discrete probability distributions, uniform distribution
- conditional probability, marginal probability
- Bayes' Theorem, prior vs posterior probability
- independent events, independent Bernoulli trials
- random variables, expected value

12 Graphs

- directed graph, undirected graph
- in-degree ($\deg^-$), out-degree ($\deg^+$), degree ($\deg$)
- walk, trail, path, circuit, cycle
- strongly connected; connected, connected component
- complete graph (K_n), ~~cyclic graph, wheel, n-cube~~
- bipartite graphs, complete bipartite graph ($K_{m,n}$)
- ~~subgraph, graph union~~
- graph isomorphism
- ~~graph representations~~
- tree, forest
- spanning tree
- rooted tree, ancestor, descendant, parent, child

General Advice

- Read carefully.
  - Make sure you gather all of the relevant information.
  - Make sure you answer all of the parts of the question.
- When answering a yes/no question, start your answer with the word “yes” or “no”, then elaborate if needed.
- Write to convince me of your understanding.
- Give specific answers.

  This class is about *abstractions*, but the utility of an abstraction is the ability to apply it to specific circumstances.
- Be careful. Check your work.

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Good luck!