1 Administrative

- Homework 01 questions on Piazza
- Midterm exam(s):
  - probably two midterms
  - first probably in second half of October

2 From last time

Calculating asymptotic bounds from recurrence equations

- recursion trees
- the master method / theorem

Introduced heaps

- preheap: binary tree with each level filled except maybe last, last is filled from left
- heap: a preheap, plus:
  each node’s key is greater than its children’s keys

3 Building a heap

(resume at slide 14)
3.1 Slide 15: Heapify
Why must we exchange with the larger child node? That is, why can’t we pick either child to exchange and recur on?

3.2 Slide 18: Heapify run time

Q1: Why \(2^{n}/3\)?

- if the right subtree has \(2^{k}\) nodes
- left subtree can have at most \(2^{k+1}\)
- total is \(2^{k} + 2^{k+1} = 3 \cdot 2^{k}\)

Q2: Does it matter?
\[ p = \log_{2/3}{1} = 0 \]
In general, \(\log_{b}{1} = 0\)

3.2.1 Heapify correctness

Heapify(A,i) correctness conditions:
- Preconditions:
  - The tree rooted at \(\text{Left}(i)\), if it exists, is a heap.
  - The tree rooted at \(\text{Right}(i)\), if it exists, is a heap.
- Postcondition:
  - The tree rooted at \(i\) is a heap.

3.3 Slide 19: Building a heap

Alternate version with simpler loop bounds:

BuildHeap(A) :=

\[
\text{heapsize}[A] \leftarrow \text{length}[A]
\text{for } i \leftarrow \text{length}[A] \text{ to } 1 \text{ do}
\quad \text{Heapify}(A, i)
\text{end for}
\]

Why is this the same?

3.4 Slide 21: Running time for BuildHeap

See Appendix A, specifically A.8 for more information about the convergence of the infinite series.