Analysis of Algorithms
Outline

1 Performance

2 Time Complexity

3 Space Complexity
Performance

Algorithms are methods for solving computational problems.

Data structures are schemes for arranging data, amenable to efficient processing by algorithms.

The performance characteristics of a program is determined by:
- its time complexity, i.e., how long it takes;
- its space complexity, i.e., how much memory it needs.

The execution time of a program of size $n$ is a function $f(n)$ determined from the cost of executing each statement, and the frequency of execution of each statement.

The running time $T(n)$ of the program is an approximation of $f(n)$ obtained by ignoring any lower-order terms and constant coefficients.

For example, if $f(n) = 3n^2 + 78n + 42$, then $T(n) = n^2$. 
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For example, if $f(n) = 31n^2 + 78n + 42$, then $T(n) = n^2$
Program: triplesum.py

• Command-line input: filename (String)
• Standard output: the number of unordered triples \((x, y, z)\) from the file such that \(x + y + z = 0\)

```
$ cat ../data/1Kints.txt
324110
-442472
...
745942

$ /usr/bin/time --format='%e seconds' python3 triplesum.py ../data/1Kints.txt
70.0.7 seconds

$ /usr/bin/time --format='%e seconds' python3 triplesum.py ../data/2Kints.txt
528.5.9 seconds
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Program: triplesum.py
Time Complexity

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```
> ~/workspace/dsa/programs
$ cat ../../../data/1Kints.txt
  324110
  -442472
  ...
  745942
$ /usr/bin/time --format='%e seconds' python3 triplesum.py ../../../data/1Kints.txt
  70
  0.7 seconds
$ /usr/bin/time --format='%e seconds' python3 triplesum.py ../../../data/2Kints.txt
  528
  5.9 seconds
```
```python
from instream import InStream
import stdio
import sys

def main():
    inStream = InStream(sys.argv[1])
    a = inStream.readInts()
    stdio.writeln(count(a))

def count(a):
    n = len(a)
    count = 0
    for i in range(0, n):
        for j in range(i + 1, n):
            for k in range(j + 1, n):
                if a[i] + a[j] + a[k] == 0:
                    count += 1
    return count

if __name__ == '__main__':
    main()
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Time Complexity

\[ f(n) = 0.2273121n^3 + 0.007625303n^2 + 0.006868505n + 0.01817256 \]

\[ T(n) = n^3 \]
Time Complexity

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
</tr>
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<tbody>
<tr>
<td>1K</td>
<td>0.28s</td>
</tr>
<tr>
<td>2K</td>
<td>1.8s</td>
</tr>
<tr>
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$$f(n) = 0.2273121n^3 + 0.007625303n^2 + 0.006868505n + 0.01817256$$

$$T(n) = n^3$$
def count(a):
    n = len(a)
    count = 0
    for i in range(0, n):
        for j in range(i + 1, n):
            for k in range(j + 1, n):
                if a[i] + a[j] + a[k] == 0:
                    count += 1
    return count

Statement Block Time Frequency Total Time
A t 4 1 t 4
B t 3 n t 3 n
C t 2 n^2 / 2 - n / 2 t 2 (n^2 / 2 - n / 2)
D t 1 n^3 / 6 - n^2 / 2 + n / 3 t 1 (n^3 / 6 - n^2 / 2 + n / 3)
E t 0 x (depends on input) t 0 x

Grand total: \( f(n) = (t_1 / 6)n^3 + (t_2 / 2 - t_1 / 2)n^2 + (t_1 / 3 - t_2 / 2 + t_3)n + t_4 + t_0 x \)

Running time: \( T(n) = n^3 \)
### Time Complexity

```python
def count(a):
    n = len(a)
    count = 0
    for i in range(0, n):
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            for k in range(j + 1, n):
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<td>(4)</td>
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<td>(3)</td>
<td>(n)</td>
</tr>
<tr>
<td>[C]</td>
<td>(2)</td>
<td>(n^2)</td>
</tr>
<tr>
<td>[D]</td>
<td>(1)</td>
<td>(n^3)</td>
</tr>
<tr>
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<td>(x)</td>
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**Gran total:** \(f(n) = \left(\frac{t_1}{6}\right)n^3 + \left(\frac{t_2}{2} - \frac{t_1}{2}\right)n^2 + \left(\frac{t_1}{3} - \frac{t_2}{2} + t_3\right)n + t_4 + t_0x\)

**Running time:** \(T(n) = n^3\)
def count(a):
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<tr>
<td>[B]</td>
<td>$t_3$</td>
<td>$n$</td>
<td>$t_3n$</td>
</tr>
<tr>
<td>[C]</td>
<td>$t_2$</td>
<td>$\binom{n}{2}^1 = n^2/2 - n/2$</td>
<td>$t_2(n^2/2 - n/2)$</td>
</tr>
<tr>
<td>[D]</td>
<td>$t_1$</td>
<td>$\binom{n}{3} = n^3/6 - n^2/2 + n/3$</td>
<td>$t_1(n^3/6 - n^2/2 + n/3)$</td>
</tr>
<tr>
<td>[E]</td>
<td>$t_0$</td>
<td>$\times$ (depends on input)</td>
<td>$t_0 \times$</td>
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\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}
\]
**Time Complexity**

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def count(a):
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Running time: $T(n) = n^3$

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}
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## Time Complexity

<table>
<thead>
<tr>
<th>Name</th>
<th>$T(n)$</th>
<th>Code Description</th>
<th>Example</th>
</tr>
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<tbody>
<tr>
<td>constant</td>
<td>1</td>
<td>statement</td>
<td>increment the $i$th element in an array</td>
</tr>
<tr>
<td>logarithmic</td>
<td>$\log n$</td>
<td>divide and discard</td>
<td>binary search</td>
</tr>
<tr>
<td>linear</td>
<td>$n$</td>
<td>loop</td>
<td>find the maximum</td>
</tr>
<tr>
<td>linearithmic</td>
<td>$n \log n$</td>
<td>divide and conquer</td>
<td>merge sort</td>
</tr>
<tr>
<td>quadratic</td>
<td>$n^2$</td>
<td>double loop</td>
<td>check all ordered pairs</td>
</tr>
<tr>
<td>cubic</td>
<td>$n^3$</td>
<td>triple loop</td>
<td>check all ordered triples</td>
</tr>
<tr>
<td>exponential</td>
<td>$2^n$</td>
<td>exhaustive search</td>
<td>check all subsets</td>
</tr>
</tbody>
</table>
### Running time classifications

<table>
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**Space Complexity**

The sizes of objects of built-in types differ from system to system, so the sizes of data types that we create also differ accordingly.

The function call `sys.getsizeof(x)` returns the number of bytes that a built-in object `x` consumes on a particular system.

Sizes of built-in objects on a typical system:

<table>
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<th>Object Type</th>
<th>Size in Bytes</th>
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<tbody>
<tr>
<td>integer</td>
<td>24</td>
</tr>
<tr>
<td>float</td>
<td>24</td>
</tr>
<tr>
<td>boolean</td>
<td>24</td>
</tr>
<tr>
<td>string of <code>n</code> characters</td>
<td>40 + <code>n</code></td>
</tr>
<tr>
<td>list of <code>n</code> integers</td>
<td>72 + 8 + 8<code>n</code> + 24</td>
</tr>
<tr>
<td><code>m</code>-by-<code>n</code> list of integers</td>
<td>72 + 32<code>m</code> + 80<code>m</code> + 32<code>mn</code></td>
</tr>
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User-defined objects can be hundreds of bytes, at least.
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<td>string of ( n ) characters</td>
<td>( 40 + n )</td>
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<tr>
<td>list of ( n ) integers</td>
<td>( 72 + 8n + 24n = 72 + 32n )</td>
</tr>
<tr>
<td>( m )-by-( n ) list of integers</td>
<td>( 72 + 8m + m(72 + 32n) = 72 + 80m + 32mn )</td>
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