Introduction to Programming in Python

Building a Computer: Logic Circuits

Outline

Boolean Functions

Boolean Functions

A boolean variable is a variable that has the value 1 (True) or 0 (False)

Boolean Functions

A boolean variable is a variable that has the value 1 (True) or 0 (False)

A boolean function is an algebraic expression consisting of boolean variables and logical operations

Boolean Functions

A boolean variable is a variable that has the value 1 (True) or 0 (False)

A boolean function is an algebraic expression consisting of boolean variables and logical operations

The three basic boolean functions: $\texttt{not}(x) = \bar{x}$, $\texttt{or}(x, y) = x + y$, and $\texttt{and}(x, y) = x \cdot y$

Boolean Functions

Boolean Functions

The truth table for a boolean function is a listing of all possible input values, together with the output value

Boolean Functions

The truth table for a boolean function is a listing of all possible input values, together with the output value

Truth tables for `not`, `or`, and `and` functions

x	$\bar{x}$
0	1
1	0

x	y	$x + y$
0	0	0
0	1	1
1	0	1
1	1	1

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

Boolean Functions

Boolean Functions

Any boolean function can be expressed in terms of the basic boolean functions using the Minterm Expansion Algorithm

Boolean Functions

Any boolean function can be expressed in terms of the basic boolean functions using the Minterm Expansion Algorithm

The algorithm

Boolean Functions

Any boolean function can be expressed in terms of the basic boolean functions using the Minterm Expansion Algorithm

The algorithm

1. Write down the truth table for the boolean function

Boolean Functions

Any boolean function can be expressed in terms of the basic boolean functions using the Minterm Expansion Algorithm

The algorithm

1. Write down the truth table for the boolean function
2. Delete all rows from the truth table where the value of the function is 0

Boolean Functions

Any boolean function can be expressed in terms of the basic boolean functions using the Minterm Expansion Algorithm

The algorithm

1. Write down the truth table for the boolean function
2. Delete all rows from the truth table where the value of the function is 0
3. For each remaining row, create a "minterm" as follows

Boolean Functions

Any boolean function can be expressed in terms of the basic boolean functions using the Minterm Expansion Algorithm

The algorithm

1. Write down the truth table for the boolean function
2. Delete all rows from the truth table where the value of the function is 0
3. For each remaining row, create a "minterm" as follows
    a. For each variable x: if its value in that row is 1, write x; otherwise, write $\bar{x}$

Boolean Functions

Any boolean function can be expressed in terms of the basic boolean functions using the Minterm Expansion Algorithm

The algorithm

1. Write down the truth table for the boolean function
2. Delete all rows from the truth table where the value of the function is 0
3. For each remaining row, create a "minterm" as follows
    a. For each variable x: if its value in that row is 1, write x; otherwise, write $\bar{x}$
    b. Combine all of the variables using $\cdot$

Boolean Functions

Any boolean function can be expressed in terms of the basic boolean functions using the Minterm Expansion Algorithm

The algorithm

1. Write down the truth table for the boolean function
2. Delete all rows from the truth table where the value of the function is 0
3. For each remaining row, create a "minterm" as follows
    a. For each variable x: if its value in that row is 1, write x; otherwise, write $\bar{x}$
    b. Combine all of the variables using $\cdot$
4. Combine all of the minterms using $+$

Boolean Functions

Boolean Functions

Example: consider the proposition “if you score over 93% in this course, then you will get an A”

Boolean Functions

Example: consider the proposition "if you score over 93% in this course, then you will get an A"

The proposition is described by the `implication` function ($x \implies y$)

x	y	$x \implies y$	minterm
0	0	1	
0	1	1	
1	0	0	
1	1	1	

Boolean Functions

Example: consider the proposition "if you score over 93% in this course, then you will get an A"

The proposition is described by the `implication` function ($x \implies y$)

x	y	$x \implies y$	minterm
0	0	1	
0	1	1	
1	0	0	
1	1	1	

Example: consider the proposition "if you score over 93% in this course, then you will get an A"

The proposition is described by the `implication` function ($x \implies y$)

x	y	x $\implies$ y	minterm
0	0	1	$\bar{x} \cdot \bar{y}$
0	1	1	
1	0	0	
1	1	1	

Boolean Functions

Example: consider the proposition "if you score over 93% in this course, then you will get an A"

The proposition is described by the `implication` function ($x \implies y$)

x	y	$x \implies y$	minterm
0	0	1	$\bar{x} \cdot \bar{y}$
0	1	1	$\bar{x} \cdot y$
1	0	0	
1	1	1	

Example: consider the proposition "if you score over 93% in this course, then you will get an A"

The proposition is described by the `implication` function ($x \implies y$)

x	y	$x \implies y$	minterm
0	0	1	$\bar{x} \cdot \bar{y}$
0	1	1	$\bar{x} \cdot y$
1	0	0	
1	1	1	$x \cdot y$

Example: consider the proposition "if you score over 93% in this course, then you will get an A"

The proposition is described by the `implication` function ($x \implies y$)

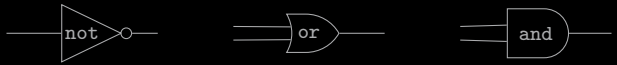
x	y	$x \implies y$	minterm
0	0	1	$\bar{x} \cdot \bar{y}$
0	1	1	$\bar{x} \cdot y$
1	0	0	
1	1	1	$x \cdot y$

Therefore, $\texttt{implication}(x, y) = \bar{x} \cdot \bar{y} + \bar{x} \cdot y + x \cdot y$

Logic Circuits

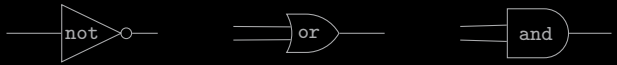
Logic Circuits

The circuits (called gates) that implement the `not`, `or`, and `and` functions

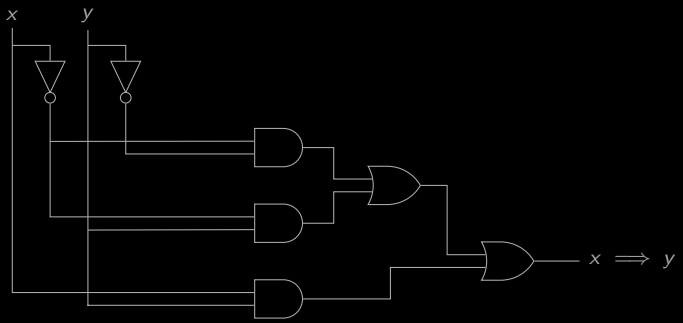


Logic Circuits

The circuits (called gates) that implement the `not`, `or`, and `and` functions

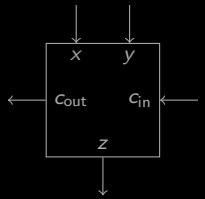


The circuit for the `implication` function $\bar{x} \cdot \bar{y} + \bar{x} \cdot y + x \cdot y$



Logic Circuits

A full adder (FA) circuit can add two 1-bit numbers (with carry) to produce a 2-bit result



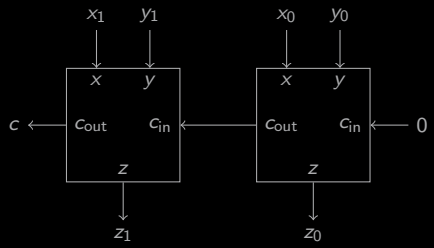
x	y	c_{in}	z	c_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Logic Circuits

An n-bit ripple-carry adder for adding two n-bit numbers is n FA circuits chained together

An n-bit ripple-carry adder for adding two n-bit numbers is n FA circuits chained together

Example (2-bit ripple-carry adder for adding two 2-bit numbers)



Logic Circuits

Logic Circuits

Truth table for a `nor` gate (`or` followed by `not`)

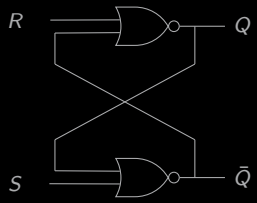
x	y	$\overline{x + y}$
0	0	1
0	1	0
1	0	0
1	1	0

Logic Circuits

Truth table for a `nor` gate (`or` followed by `not`)

x	y	$\overline{x + y}$
0	0	1
0	1	0
1	0	0
1	1	0

A 1-bit memory circuit, called a latch, built using two `nor` gates



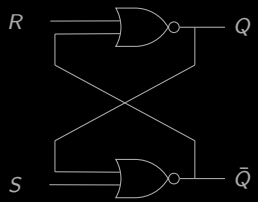
S	R	Q	$\bar{Q}$
0	0	0	1
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1

Logic Circuits

Truth table for a `nor` gate (`or` followed by `not`)

x	y	$\overline{x + y}$
0	0	1
0	1	0
1	0	0
1	1	0

A 1-bit memory circuit, called a latch, built using two `nor` gates



S	R	Q	$\bar{Q}$
0	0	0	1
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1

Billion latches can be combined to produce a 1GB Random Access Memory (RAM) module