1 Exercises

Exercise 1. Consider the following recursive function:

```python
def mystery(a, b):
    if b == 0:
        return 0
    if a == 0:
        return mystery(b - 1, a)
    return b + mystery(b, a - 1)
```

a. What is the value returned by the call `mystery(10, 0)`?
b. What is the value returned by the call `mystery(0, 10)`?
c. What is the value returned by the call `mystery(3, 7)`?
d. What is the value returned by the call `mystery(10, 3)`?
e. What is the value returned by the call `mystery(200, 300)`?
f. What does the function `mystery()` compute in general about `a` and `b`?

Exercise 2. Consider the function \( S(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2 \), where \( n \) is a positive integer.

a. What is the value of \( S(5) \)?
b. Provide a recursive definition for \( S(n) \).
c. Implement a function \( S(n) \) using recursion, such that it computes and returns \( S(n) \)
d. Trace the function call \( S(5) \).

2 Solutions

Solution 1.

a. 0
b. 0
c. 21
d. 30
e. 60000
f. The product \( ab \).

Solution 2.

a. 55
b. \( S(n) = \begin{cases} 
    n^2 + S(n - 1) & \text{if } n > 1, \\
    1 & \text{if } n = 1.
\end{cases} \)
c.

```python
def S(n):
    if n == 1:
        return 1
    return n * n + S(n - 1)
```
d.

\[
\begin{align*}
S(5) \\
S(4) \\
S(3) \\
S(2) \\
S(1) \\
&\quad \text{return 1} \\
&\quad \text{return 2 \times 2 + 1 = 5} \\
&\quad \text{return 3 \times 3 + 5 = 14} \\
&\quad \text{return 4 \times 4 + 14 = 30} \\
&\quad \text{return 5 \times 5 + 30 = 55}
\end{align*}
\]