Exercise 1. Consider an array `a` with $n = 10^4$ integers.

a. Roughly how many comparisons are involved if one performs $m = 10^6$ linear search operations on `a`?

b. Roughly how many comparisons (sorting and searching included) are involved if one performs $m = 10^6$ binary search operations on `a`?

Exercise 2. Consider the following table, which gives the running time $T(n)$ for a program for various input sizes n:

n	$T(n)$
100	3s
200	25s
400	200s
800	1,599s

What is the functional form of $T(n)$?

Exercise 3. What is the running time classification (constant, logarithmic, linear, linearithmic, quadratic, cubic, or exponential) for each of the following tasks:

a. Adding two $n \times n$ matrices.

b. Enumerating the subsets of a set of n items.

c. Finding the average of n numbers.

d. Counting the unordered triples (a, b, c) in an array of n positive integers such that $a^2 + b^2 = c^2$.

e. Searching for a key in a sorted array of n keys.

f. Printing the ith element in an array of size n.

g. Adding up the diagonal elements of an $n \times n$ matrix.

h. Counting the unordered doubles (a, b) in an array of n integers such that $a + b = 0$.

Exercise 4. What is the running time $T(n)$ for each of the following code fragments:

a.

```
int sum = 0;
for (int i = n; i > 0; i /= 2) {
   for (int j = 0; j < i; j++) {
      sum++;
   }
}
```

b.

```
int sum = 0;
for (int i = 1; i < n; i *= 2) {
   for(int j = 0; j < i; j++) {
      sum++;
   }
}
```

c.

```
int sum = 0;
for (int i = 1; i < n; i *= 2) {
   for (int j = 0; j < n; j++) {
      sum++;
   }
}
```

Exercise 5. Consider a data type `Planet` with the attributes `String name` and `int moons`. What is the memory footprint (in bytes) of the array `planets`, created and initialized in the following manner?

```
Planet[] planets = new Planet[8];
planets[0] = new Planet("Mercury", 0);
planets[1] = new Planet("Venus", 0);
planets[2] = new Planet("Earth", 1);
planets[3] = new Planet("Mars", 2);
planets[4] = new Planet("Jupiter", 67);
planets[5] = new Planet("Saturn", 62);
planets[6] = new Planet("Uranus", 27);
planets[7] = new Planet("Neptune", 14);
```

Exercise 6. Consider the following function:

```
public static int mystery(Node<Integer> first) {
    int x = 0;
    for (Node y = first, int i = 0; y != null; y = y.next, i++) {
        x += (i % 2 == 0) ? y.item : 0;
    }
    return x;
}
```

a. What does `mystery()` compute and return in general?

b. What will `mystery()` return if the argument `a` represents a linked list containing integers $1, 2, 3, \ldots, 10$?

Exercise 7. Consider the following function:

```
public static int mystery(Bag<Integer> bag) {
    int x = 0;
    Itereator<Integer> iter = bag.iterator();
    while (iter.hasNext()) {
        x += iter.next();
    }
    return x;
}
```

a. What does `mystery()` compute and return in general?

b. What will `mystery()` return if the argument `a` represents a bag containing the integers $1, 2, 3, \ldots, 10$?

Exercise 8. Suppose that a minus sign in the input indicates pop the stack and write the returned value to standard output, and any other string indicates push the string onto the stack. Further suppose that following input is processed:

```
it was - the best - of times - - - it was - the - - worst - of times -
```

a. What is written to standard output?

b. What are the contents (from top to bottom) left on the stack?

Exercise 9. Suppose that an intermixed sequence of (stack) push and pop operations are performed. The pushes push the integers 0 through 9 in order; the pops print out the return value. Which of the following sequence(s) could not occur?

A. `4 3 2 1 0 9 8 7 6 5`

B. `4 6 8 7 5 3 2 9 0 1`

C. `2 5 6 7 4 8 9 3 1 0`

D. `4 3 2 1 0 5 6 7 8 9`

E. `1 2 3 4 5 6 9 8 7 0`

F. `0 4 6 5 3 8 1 7 2 9`

G. `1 4 7 9 8 6 5 3 0 2`

H. `2 1 4 3 6 5 8 7 9 0`

Exercise 10. Consider the following code fragment:

```
Stack<Integer> s = new Stack<Integer>();
while (n > 0) {
   s.push(n % 2);
   n = n / 2;
}
while (!s.isEmpty()) {
    StdOut.print(s.pop());
}
StdOut.println();
```

a. What does the code output when `n` is 50?

b. What does the code output in general for a non-negative integer `n`?

Exercise 11. Suppose that a minus sign in the input indicates dequeue the queue and write the return value to standard output, and any other string indicates enqueue the string onto the queue. Further suppose that following input is processed:

`it was - the best - of times - - - it was - the - - worst - of times -`

a. What is written to standard output?

b. What are the contents (from front to back) left on the queue?

Exercise 12. Suppose that a client performs an intermixed sequence of (queue) enqueue and dequeue operations. The enqueue operations put the integers 0 through 9 in order onto the queue; the dequeue operations print out the return value. Which of the following sequence(s) could not occur?

A. `0 1 2 3 4 5 6 7 8 9`

B. `4 6 8 7 5 3 2 9 0 1`

C. `2 5 6 7 4 8 9 3 1 0`

D. `4 3 2 1 0 5 6 7 8 9`

Exercise 13. What does the following code fragment do to the queue `q`?

```java
Stack<String> s = new Stack<String>();
while(!q.isEmpty()) {
   s.push(q.dequeue());
}
while(!s.isEmpty()) {
   q.enqueue(s.pop());
}
```

Exercise 14. Suppose we are using the UF data structure to solve the dynamic connectivity problem with 10 sites and input pairs $(1, 2)$, $(7, 8)$, $(1, 6)$, $(0, 5)$, $(3, 8)$, $(2, 3)$, $(6, 7)$, $(2, 7)$, and $(4, 9)$, arriving in that order.

a. How many components does UF identify?

b. What are those components?

c. Will the number of components or their membership change if the input pairs arrive in a different order than above?

d. Suppose we process the pairs using `QuickFindUF`. What are the values in the `id` array after all the pairs are processed?

e. Suppose we process the pairs using `QuickUnionUF`. What are the values in the `parent` array after all the pairs are processed?

f. Suppose we process the pairs using `WeightedQuickUnionUF`. What are the values in the `parent` and `size` arrays after all the pairs are processed?

Solutions

Solution 1.

a. $10^6 \cdot 10^4 = 10^{10}$

b. $10^4 \log 10^4$ (sorting) $+\ 10^6 \log 10^4$ (searching)

Solution 2. $T(n) = n^3$ (cubic)

Solution 3.

a. Quadratic

b. Exponential

c. Linear

d. Cubic

e. Logarithmic

f. Constant

g. Linear

h. Quadratic

Solution 4.

a. $T(n) = n$ (linear)

b. $T(n) = n$ (linear)

c. $T(n) = n \log n$ (linearithmic)

Solution 5. $8 \times 12 + 2 \times (7 + 5 + 5 + 4 + 7 + 6 + 6 + 7) = 190$ bytes

Solution 6.

a. Computes and returns the sum of every other integer in `node`, starting at the first.

b. `25`

Solution 7.

a. Computes and returns the sum of the integers in `bag`.

b. `55`

Solution 8.

a. `was best times of the was the it worst times`

b. `of it`

Solution 9. `B`, `F`, and `G`

Solution 10.

a. `110010`

b. Prints the binary representation of `n`.

Solution 11.

a. `it was the best of times it was the worst`

b. `of times`

Solution 12. `B`, `C`, and `D`

Solution 13. Reverses the items on the queue.

Solution 14.

a. 3

b. $\{0, 5\}, \{4, 9\},$ and $\{1, 2, 3, 6, 7, 8\}$

c. No

d. `id = {5, 8, 8, 8, 9, 5, 8, 8, 8, 9}`

e. `parent = {5, 2, 6, 8, 9, 5, 8, 8, 8, 9}`

f. `parent = {0, 1, 1, 7, 4, 0, 1, 1, 7, 4}, size = {2, 6, 1, 1, 2, 1, 1, 3, 1, 1}`