1 Exercises

**Exercise 1.** Consider an array $a$ with $n = 10^4$ integers.

a. Roughly how many comparisons are involved if one performs $m = 10^6$ linear search operations on $a$?

b. Roughly how many comparisons (sorting and searching included) are involved if one performs $m = 10^6$ binary search operations on $a$?

**Exercise 2.** Consider the following table, which gives the running time $T(n)$ for a program for various input sizes $n$:

<table>
<thead>
<tr>
<th>$n$</th>
<th>$T(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>3s</td>
</tr>
<tr>
<td>200</td>
<td>25s</td>
</tr>
<tr>
<td>400</td>
<td>200s</td>
</tr>
<tr>
<td>800</td>
<td>1,599s</td>
</tr>
</tbody>
</table>

What is the functional form of $T(n)$?

**Exercise 3.** What is the running time classification (constant, logarithmic, linear, linearithmic, quadratic, cubic, or exponential) for each of the following tasks:

a. Adding two $n \times n$ matrices.

b. Enumerating the subsets of a set of $n$ items.

c. Finding the average of $n$ numbers.

d. Counting the unordered triples $(a, b, c)$ in an array of $n$ positive integers such that $a^2 + b^2 = c^2$.

e. Searching for a key in a sorted array of $n$ keys.

f. Printing the $i$th element in an array of size $n$.

g. Adding up the diagonal elements of an $n \times n$ matrix.

h. Counting the unordered doubles $(a, b)$ in an array of $n$ integers such that $a + b = 0$.

**Exercise 4.** What is the running time $T(n)$ for each of the following code fragments:

a.

```c
int sum = 0;
for (int i = n; i > 0; i /= 2) {
    for (int j = 0; j < i; j++) {
        sum++;
    }
}
```

b.

```c
int sum = 0;
for (int i = 1; i < n; i *= 2) {
    for(int j = 0; j < i; j++) {
        sum++;
    }
}
```
c.
```java
int sum = 0;
for (int i = 1; i < n; i *= 2) {
    for (int j = 0; j < n; j++) {
        sum++;
    }
}
```

**Exercise 5.** Consider a data type `Planet` with the attributes `String name` and `int moons`. What is the memory footprint (in bytes) of the array `planets`, created and initialized in the following manner?

```java
Planet[] planets = new Planet[8];
planets[0] = new Planet("Mercury", 0);
planets[1] = new Planet("Venus", 0);
planets[2] = new Planet("Earth", 1);
planets[3] = new Planet("Mars", 2);
planets[4] = new Planet("Jupiter", 67);
planets[5] = new Planet("Saturn", 62);
planets[6] = new Planet("Uranus", 27);
planets[7] = new Planet("Neptune", 14);
```

## 2 Solutions

**Solution 1.**

a. 10^6 \cdot 10^4 = 10^{10}

b. 10^4 \log_{10} 10^4 \text{ (sorting)} + 10^6 \log_{10} 10^4 \text{ (searching)}

**Solution 2.** $T(n) = n^3$ (cubic)

**Solution 3.**

a. Quadratic

b. Exponential

c. Linear

d. Cubic

e. Logarithmic

f. Constant

g. Linear

h. Quadratic

**Solution 4.**

a. $T(n) = n$ (linear)

b. $T(n) = n$ (linear)

c. $T(n) = n \log n$ (linearithmic)

**Solution 5.** $8 \times 12 + 2 \times (7 + 5 + 5 + 4 + 7 + 6 + 6 + 7) = 190$ bytes