

1 Exercises

Exercise 1. Consider an array \mathbf{a} with $n = 10^4$ integers.

- a. Roughly how many comparisons are involved if one performs $m = 10^6$ linear search operations on \mathbf{a} ?
- b. Roughly how many comparisons (sorting and searching included) are involved if one performs $m = 10^6$ binary search operations on \mathbf{a} ?

Exercise 2. Consider the following table, which gives the running time $T(n)$ for a program for various input sizes n :

n	$T(n)$
100	3s
200	25s
400	200s
800	1,599s

What is the functional form of $T(n)$?

Exercise 3. What is the running time classification (constant, logarithmic, linear, linearithmic, quadratic, cubic, or exponential) for each of the following tasks:

- a. Adding two $n \times n$ matrices.
- b. Enumerating the subsets of a set of n items.
- c. Finding the average of n numbers.
- d. Counting the unordered triples (a, b, c) in an array of n positive integers such that $a^2 + b^2 = c^2$.
- e. Searching for a key in a sorted array of n keys.
- f. Printing the i th element in an array of size n .
- g. Adding up the diagonal elements of an $n \times n$ matrix.
- h. Counting the unordered doubles (a, b) in an array of n integers such that $a + b = 0$.

Exercise 4. What is the running time $T(n)$ for each of the following code fragments:

a.

```
int sum = 0;
for (int i = n; i > 0; i /= 2) {
    for (int j = 0; j < i; j++) {
        sum++;
    }
}
```

b.

```
int sum = 0;
for (int i = 1; i < n; i *= 2) {
    for (int j = 0; j < i; j++) {
        sum++;
    }
}
```

c.

```
int sum = 0;
for (int i = 1; i < n; i *= 2) {
    for (int j = 0; j < n; j++) {
        sum++;
    }
}
```

Exercise 5. Consider a data type `Planet` with the attributes `String name` and `int moons`. What is the memory footprint (in bytes) of the array `planets`, created and initialized in the following manner?

```
Planet[] planets = new Planet[8];
planets[0] = new Planet("Mercury", 0);
planets[1] = new Planet("Venus", 0);
planets[2] = new Planet("Earth", 1);
planets[3] = new Planet("Mars", 2);
planets[4] = new Planet("Jupiter", 67);
planets[5] = new Planet("Saturn", 62);
planets[6] = new Planet("Uranus", 27);
planets[7] = new Planet("Neptune", 14);
```

2 Solutions

Solution 1.

- a. $10^6 \cdot 10^4 = 10^{10}$
- b. $10^4 \log 10^4$ (sorting) + $10^6 \log 10^4$ (searching)

Solution 2. $T(n) = n^3$ (cubic)

Solution 3.

- a. Quadratic
- b. Exponential
- c. Linear
- d. Cubic
- e. Logarithmic
- f. Constant
- g. Linear
- h. Quadratic

Solution 4.

- a. $T(n) = n$ (linear)
- b. $T(n) = n$ (linear)
- c. $T(n) = n \log n$ (linearithmic)

Solution 5. $8 \times 12 + 2 \times (7 + 5 + 5 + 4 + 7 + 6 + 6 + 7) = 190$ bytes