## Name:

## Instructions

1. Write your name at the top of the first page and your initials at the bottom of every page.
2. When you are done, return the exam with all the pages arranged in ascending order. Do not staple the exam.
3. This is a closed-book exam. No form of communication is permitted (eg, talking, texting, etc.), except with the course staff.
4. No electronic devices are permitted.
5. There are 25 multiple-choice questions in this exam, each worth 3 points.
6. The answer to each question must be marked with a pencil as shown in the following example. It will be considered incorrect otherwise.

Example. What is Albert Einstein's miracle year?
(A) 1879
(B) 1900
(C) 1905
(D) 1917
(E) 1955
7. You may use the blank spaces for any scratch work.
8. Discussing the exam contents with anyone who has not taken the exam is a violation of the academic honesty code.

Problem 1. Consider inserting the following key-value pairs in that order into a symbol table st.

$$
\begin{array}{rrrrrrrrrrrrr}
\text { key: } & \text { R } & \text { Q } & \text { J } & \text { G } & \text { L } & \text { R } & \text { M } & \text { I } & \text { Q } & \text { H } & \text { R } & \text { V } \\
\text { value: } & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12
\end{array}
$$

a. What is the value returned by st.size()?
(A) 12
(B) 11
(C) 9
(D) 8
(E) 10
b. What is the value returned by st.get("R")?
(A) 6
(B) 11
(C) 3
(D) 18
(E) 1

Problem 2. Consider inserting the following keys (assume values to be non null and arbitrary) into a binary search tree (BST) symbol table st, an object of type вst.

$$
\begin{array}{llllllllllll}
\mathrm{G} & \mathrm{~T} & \mathrm{~J} & \mathrm{Q} & \mathrm{H} & \mathrm{Z} & \mathrm{~K} & \mathrm{~A} & \mathrm{O} & \mathrm{C} & \mathrm{M} & \mathrm{~B}
\end{array}
$$

a. What is the height of the BST (assume root to be at height 0 )?
(A) 5
(B) 7
(C) 6
(D) 4
(E) 8
b. What is the value returned by st.rank("M")?
(A) 7
(B) 5
(C) 8
(D) 6
(E) 4
c. What is the order in which the keys are visited if we traverse the BST in pre-order?





d. What is the order in which the keys are visited if we traverse the BST in in-order?

e. What is the order in which the keys are visited if we traverse the BST in post-order?


Problem 3. Consider inserting the following keys into an initially empty 2-3 search tree.

$$
\begin{array}{llllllllllll}
\text { B } & \text { Q } & \text { P } & \text { F } & \text { N } & \text { W } & \text { G } & \text { J } & \text { L } & \text { H } & \text { U } & \text { X }
\end{array}
$$

a. What is the height of the tree that results (assume root to be at height zero)?
(A) 3
(B) 5
(C) 4
(D) 1
(E) 2
b. How many nodes does the tree contain?
(A) 6
(B) 8
(C) 5
(D) 9
(E) 7
c. How many 2 -nodes does the tree contain?
(A) 4
(B) 6
(C) 5
(D) 3
(E) 7
d. How many 3 -nodes does the tree contain?
(A) 6
(B) 5
(C) 4
(D) 3
(E) 7

Problem 4. Suppose you insert the key 9 into the following left-leaning red-black BST:


Allowed operations (rotations and color flip):

a. What is the first operation that results?
(A) Rotate 8 left
(B) Rotate 10 right
(C) Rotate 12 right
(D) Rotate 6 left
(E) Color flip 9
b. What is the second operation that results?
(A) Rotate 8 left
(B) Rotate 10 right
(C) Rotate 12 right
(D) Rotate 6 left
(E) Color flip 9
c. What is the third operation that results?
(A) Rotate 8 left
(B) Rotate 12 right
(C) Rotate 10 right
(D) Rotate 6 left
(E) Color flip 9
d. What is the fourth operation that results?
(A) Rotate 8 left
(B) Rotate 6 left
(C) Rotate 12 right
(D) Rotate 10 right
(E) Color flip 9
e. What is the fifth operation that results?
(A) Rotate 12 right
(B) Rotate 6 left
(C) Rotate 10 right
(D) Color flip 9
(E) Rotate 8 left

Problem 5. Consider inserting the following keys (assume values to be non null and arbitrary) into an initially empty hash table of $M=5$ lists, using separate chaining. Use the hash function $h(k)=k \bmod M$ to transform the $k$ th letter of the alphabet into a table index, where $1 \leq k \leq 26$.
$\begin{array}{llllllllllll}\text { J } & \text { D } & \text { W } & \text { E } & \text { V } & \text { U } & \text { L } & \text { P } & \text { F } & \text { K } & \text { X } & \text { Y }\end{array}$
a. What is the length of the longest chain?
(A) 1
(B) 3
(C) 5
(D) 4
(E) 2
b. Which of the following keys is in the longest chain?
(A) U
(B) 0
(C) $v$
(D) J
(E) w

Problem 6. Perform a depth-first search in the digraph below, starting from vertex 0 . Assume the adjacency lists are in sorted order: for example, when iterating over the edges pointing from 3 , process the edge $3 \rightarrow 2$ before either $3 \rightarrow 7$ or $3 \rightarrow 8$.

a. List all vertices in pre-order.
(A) 0937841265
(B) 0936154782
(C) 0932176845
(D) 0938264517
(E) 0932461587
b. List all vertices in post-order.
(A) 5862147390
(B) 8725416390
(C) 4156872390
(D) 1687254390
(E) 4561872390
c. List all vertices in reverse post-order.
(A) 0934527861
(B) 0937412685
(C) 0932781654
(D) 0932786514
(E) 0936145278

Problem 7. Consider the following edge-weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.

a. What is the last edge that is added to the minimum spanning tree (MST) by Kruskal's algorithm?
(A) 10
(B) 16
(C) 14
(D) 12
(E) 8
b. What is the weight of the MST?
(A) 45
(B) 48
(C) 36
(D) 50
(E) 56

Problem 8. Suppose that after running Dijkstra's algorithm on an edge-weighted digraph, starting from vertex o, the values in the distTo and edgeTo arrays are as shown below.

| v | distTo[v] | edgeTo[v] |
| :---: | :---: | :---: |
| 0 | 0 | null |
| 1 | 13 | $6 \rightarrow 1$ |
| 2 | 6 | $0 \rightarrow 2$ |
| 3 | 18 | $9 \rightarrow 3$ |
| 4 | 6 | $0 \rightarrow 4$ |
| 5 | 9 | $4 \rightarrow 5$ |
| 6 | 10 | $4 \rightarrow 6$ |
| 7 | 9 | $4 \rightarrow 7$ |
| 8 | 7 | $6 \rightarrow 9$ |
| 9 | 14 | $6 \rightarrow 10$ |
| 10 | 14 |  |

a. What is the shortest path to vertex 3 ?
(A) $0 \rightarrow 4$ l> 6 -> $9 \rightarrow 1$ l> 3
(B) $0 \rightarrow 4$-> $8 \rightarrow 1 \rightarrow 3$
(C) $0 \rightarrow 10 \rightarrow 1 \rightarrow 7 \rightarrow 3$
(D) $0 \rightarrow 4$-> 6 -> $9 \rightarrow 6$-> 3
(E) 0 -> 4 -> 6 -> 9 -> 3
b. What is the weight on the edge $6 \rightarrow 9$ ?
(A) 8
(B) 2
(C) 6
(D) 4
(E) 10

## Answers

Problem 1. C, B
Problem 2. C, A, C, A, D
Problem 3. E, B, A, C
Problem 4. A, B, E, B, A
Problem 5. D, A
Problem 6. C, D, A
Problem 7. C, A
Problem 8. E, D

