Name:

	Instructions						
1.	Write your name at the top of the <i>first</i> page and your initials at the bottom of <i>every</i> page.						
2.	When you are done, return the exam with <i>all</i> the pages arranged in <i>ascending</i> order. Do <i>not</i> staple the exam.						
3.	3. This is a closed-book exam. No form of communication is permitted (eg, talking, texting, etc.), except with the course staff.						
4.	No electronic devices are permitted.						
5.	There are 25 multiple-choice questions in this exam, each worth 3 points.						
6.	The answer to each question must be marked <i>with a pencil</i> as shown in the following example. It will be considered incorrect otherwise.						
	Example. What is Albert Einstein's miracle year?						
	A 1879						
	B 1900						
	C 1905						
	D 1917						
	(E) 1955						
7.	You may use the blank spaces for any scratch work.						

8. Discussing the exam contents with anyone who has not taken the exam is a violation of the academic honesty code.

Problem 1. Consider inserting the following key-value pairs in that order into a symbol table st.

V key: Q J G L М Ι Q Η R R R value: 1 2 3 4 5 6 7 8 9 10 11 12

a. What is the value returned by st.size()?



b. What is the value returned by st.get("R")?

(A) 6
(B) 11
(C) 3
(D) 18
(E) 1

Problem 2. Consider inserting the following keys (assume values to be non null and arbitrary) into a binary search tree (BST) symbol table st, an object of type BST.

G T J Q H Z K A O C M B

a. What is the height of the BST (assume root to be at height 0)?

 A
 5

 B
 7

 C
 6

 D
 4

 E
 8

b. What is the value returned by st.rank("M")?

(\mathbf{A})	7
B	5
\bigcirc	8
\bigcirc	6
(E)	4

c. What is the order in which the keys are visited if we traverse the BST in pre-order?

\bigcirc	A	В	С	G	Н	J	К	М	0	Q	Т	Z
В	G	A	С	В	Т	J	Н	Q	0	K	Z	М
\bigcirc	G	A	С	В	Т	J	Н	Q	К	0	М	Z
\bigcirc	В	С	A	Н	М	0	К	Q	J	Z	Т	G
(E)	G	A	С	В	Т	J	Н	М	K	Z	0	Q

d. What is the order in which the keys are visited if we traverse the BST in in-order?

(A)A В Ζ С G Н J Κ М 0 Q Т (B)A В С G Н J Κ М Ζ Q 0 Т $\left[C \right]$ А В С G Н Ζ М J Κ Q Т 0 (D)В С G Н М 0 Κ Z Т A Q J (E)G A С В Т J Н Q Κ 0 М Z e. What is the order in which the keys are visited if we traverse the BST in post-order? (\mathbf{A}) В С A Н М 0 Κ Ζ Q G Т J (B)В С Н Α М 0 Κ J G Q Ζ Т C С G Ζ А В Н 0 Q Т J Κ М D В С Н 0 Ζ G А М Κ Q J Т (E)G A С В Т J Н Q К 0 М Z

Problem 3. Consider inserting the following keys into an initially empty 2-3 search tree.

BQPFNWGJLHUX

- a. What is the height of the tree that results (assume root to be at height zero)?
 - (A) 3
 (B) 5
 (C) 4
 (D) 1
 - (E) 2
- b. How many nodes does the tree contain?
 - (A) 6
 (B) 8
 (C) 5
 - (D) 9
 - (E) 7

- c. How many 2-nodes does the tree contain?
 - $\begin{array}{c} (A) & 4 \\ \hline (B) & 6 \\ \hline (C) & 5 \\ \hline (D) & 3 \\ \hline (E) & 7 \end{array}$

d. How many 3-nodes does the tree contain?

- (E) 7





Allowed operations (rotations and color flip):



- a. What is the *first* operation that results?
 - A Rotate 8 left
 B Rotate 10 right
 C Rotate 12 right
 D Rotate 6 left
 E Color flip 9

b. What is the *second* operation that results?

\bigcirc	Rotate 8 left
B	Rotate 10 right
\bigcirc	Rotate 12 right
\bigcirc	Rotate 6 left
(E)	Color flip 9

- c. What is the *third* operation that results?
 - $egin{array}{c} A & ext{Rotate 8 left} \ \hline B & ext{Rotate 12 right} \end{array}$
 - B Rotate 12 right
 - C Rotate 10 right
 - $\left(D \right)$ Rotate 6 left
 - $\left(\mathrm{E}
 ight)$ Color flip 9
- d. What is the *fourth* operation that results?
 - (A) Rotate 8 left
 (B) Rotate 6 left
 (C) Rotate 12 right
 (D) Rotate 10 right
 (E) Color flip 9

e. What is the *fifth* operation that results?

A Rotate 12 right
B Rotate 6 left
C Rotate 10 right
D Color flip 9
E Rotate 8 left

Problem 5. Consider inserting the following keys (assume values to be non null and arbitrary) into an initially empty hash table of M = 5 lists, using separate chaining. Use the hash function $h(k) = k \mod M$ to transform the kth letter of the alphabet into a table index, where $1 \le k \le 26$.

J D W E V U L P F K X Y

- a. What is the length of the longest chain?
 - (A) 1
 (B) 3
 (C) 5
 (D) 4
 (E) 2



- AUB0CVDJ
- (E) w

Problem 6. Perform a depth-first search in the digraph below, starting from vertex 0. Assume the adjacency lists are in sorted order: for example, when iterating over the edges pointing from 3, process the edge $3 \rightarrow 2$ before either $3 \rightarrow 7$ or $3 \rightarrow 8$.



- a. List all vertices in pre-order.
 - (A) 0937841265
 - (B) 0936154782
 - (C) 0932176845
 - (D) 0938264517
 - (E) 0932461587
- b. List all vertices in post-order.



- c. List all vertices in reverse post-order.
 - (A) 0 9 3 4 5 2 7 8 6 1
 (B) 0 9 3 7 4 1 2 6 8 5
 (C) 0 9 3 2 7 8 1 6 5 4
 (D) 0 9 3 2 7 8 6 5 1 4
 (E) 0 9 3 6 1 4 5 2 7 8

Problem 7. Consider the following edge-weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.



- a. What is the *last* edge that is added to the minimum spanning tree (MST) by Kruskal's algorithm?
- A) 10 В 16 С 14 D 12 E 8 b. What is the weight of the MST? A 45 В 48 С 36
 - (D) 50
 - (E) 56



v	distTo[v]	edgeTo[v]				
0	0	null				
1	13	6 -> 1				
2	6	0 -> 2 9 -> 3				
3	18					
4	6	0 -> 4				
5	9	4 -> 5 4 -> 6				
6	10					
7	9	4 -> 7				
8	7	4 -> 8				
9	14	6 -> 9				
10	14	6 -> 10				

a. What is the shortest path to vertex 3?



b. What is the weight on the edge $6 \rightarrow 9?$

A 8
B 2
C 6
D 4

(E) 10

Answers

Problem 1. C, B

Problem 2. C, A, C, A, D

Problem 3. E, B, A, C

Problem 4. A, B, E, B, A

Problem 5. D, A

Problem 6. C, D, A

Problem 7. C, A

Problem 8. E, D