Name:

**Instructions**

1. Write your name at the top of the *first* page and your initials at the bottom of *every* page.

2. When you are done, return the exam with *all* the pages arranged in *ascending* order. Do *not* staple the exam.

3. This is a closed-book exam. No form of communication is permitted (eg, talking, texting, etc.), except with the course staff.

4. No electronic devices are permitted.

5. There are 25 multiple-choice questions in this exam, each worth 3 points.

6. The answer to each question must be marked *with a pencil* as shown in the following example. It will be considered incorrect otherwise.

   **Example.** What is Albert Einstein's miracle year?

   - A 1879
   - B 1900
   - C 1905
   - D 1917
   - E 1955

7. You may use the blank spaces for any scratch work.

8. Discussing the exam contents with anyone who has not taken the exam is a violation of the academic honesty code.

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**Problem 1.** Consider inserting the following key-value pairs in that order into a symbol table st.

<table>
<thead>
<tr>
<th>key:</th>
<th>R</th>
<th>Q</th>
<th>J</th>
<th>G</th>
<th>L</th>
<th>R</th>
<th>M</th>
<th>I</th>
<th>Q</th>
<th>H</th>
<th>R</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>value:</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

a. What is the value returned by st.size()?

   - A 12
   - B 11
   - C 9
   - D 8
   - E 10

Initials: 1 / 9
b. What is the value returned by `st.get("R")`?

- A 6
- B 11
- C 3
- D 18
- E 1

Problem 2. Consider inserting the following keys (assume values to be non-null and arbitrary) into a binary search tree (BST) symbol table `st`, an object of type `BST`.

```
G T J Q H Z K A O C M B
```

a. What is the height of the BST (assume root to be at height 0)?

- A 5
- B 7
- C 6
- D 4
- E 8

b. What is the value returned by `st.rank("M")`?

- A 7
- B 5
- C 8
- D 6
- E 4

c. What is the order in which the keys are visited if we traverse the BST in pre-order?

- A A B C G H J K M O Q T Z
- B G A C B T J H Q O K Z M
- C G A C B T J H Q K O M Z
- D B C A H M O K Q J Z T G
- E G A C B T J H M K Z Q O Q
d. What is the order in which the keys are visited if we traverse the BST in in-order?

A
- A B C G H J K M O Q T Z
B
- A B C G H J K M Z Q O T
C
- A B C G H J K Q Z T O M
D
- B C A H M O K Q J Z T G
E
- G A C B T J H Q K O M Z

e. What is the order in which the keys are visited if we traverse the BST in post-order?

A
- B C A H M D K Z Q J G T
B
- B C A H M D K J G Q Z T
C
- A B C G H J K M D Q T Z
D
- B C A H M D K Q J Z T G
E
- G A C B T J H Q K O M Z

Problem 3. Consider inserting the following keys into an initially empty 2-3 search tree.

B Q P F N W G J L H U X

a. What is the height of the tree that results (assume root to be at height zero)?

A
- 3
B
- 5
C
- 4
D
- 1
E
- 2

b. How many nodes does the tree contain?

A
- 6
B
- 8
C
- 5
D
- 9
E
- 7
c. How many 2-nodes does the tree contain?

A 4
B 6
C 5
D 3
E 7

d. How many 3-nodes does the tree contain?

A 6
B 5
C 4
D 3
E 7

Problem 4. Suppose you insert the key 9 into the following left-leaning red-black BST:

Allowed operations (rotations and color flip):

a. What is the first operation that results?

A Rotate 8 left
B Rotate 10 right
C Rotate 12 right
D Rotate 6 left
E Color flip 9

Initials:
b. What is the second operation that results?

A. Rotate 8 left  
B. Rotate 10 right  
C. Rotate 12 right  
D. Rotate 6 left  
E. Color flip 9

c. What is the third operation that results?

A. Rotate 8 left  
B. Rotate 12 right  
C. Rotate 10 right  
D. Rotate 6 left  
E. Color flip 9

d. What is the fourth operation that results?

A. Rotate 8 left  
B. Rotate 6 left  
C. Rotate 12 right  
D. Rotate 10 right  
E. Color flip 9

e. What is the fifth operation that results?

A. Rotate 12 right  
B. Rotate 6 left  
C. Rotate 10 right  
D. Color flip 9  
E. Rotate 8 left

**Problem 5.** Consider inserting the following keys (assume values to be non-null and arbitrary) into an initially empty hash table of $M = 5$ lists, using separate chaining. Use the hash function $h(k) = k \mod M$ to transform the $k$th letter of the alphabet into a table index, where $1 \leq k \leq 26$.

```
J D W E V U L P F K X Y
```
a. What is the length of the longest chain?

A 1
B 3
C 5
D 4
E 2

b. Which of the following keys is in the longest chain?

A u
B 0
C v
D j
E w

Problem 6. Perform a depth-first search in the digraph below, starting from vertex 0. Assume the adjacency lists are in sorted order: for example, when iterating over the edges pointing from 3, process the edge $3 \rightarrow 2$ before either $3 \rightarrow 7$ or $3 \rightarrow 8$.

![Digraph](image)

a. List all vertices in pre-order.

A 0 9 3 7 8 4 1 2 6 5
B 0 9 3 6 1 5 4 7 8 2
C 0 9 3 2 1 7 6 8 4 5
D 0 9 3 8 2 6 4 5 1 7
E 0 9 3 2 4 6 1 5 8 7

b. List all vertices in post-order.

A 5 8 6 2 1 4 7 3 9 0
B 8 7 2 5 4 1 6 3 9 0
C 4 1 5 6 8 7 2 3 9 0
D 1 6 8 7 2 5 4 3 9 0
E 4 5 6 1 8 7 2 3 9 0

Initials: 6 / 9
c. List all vertices in reverse post-order.

A 0 9 3 4 5 2 7 8 6 1
B 0 9 3 7 4 1 2 6 8 5
C 0 9 3 2 7 8 1 6 5 4
D 0 9 3 2 7 8 6 5 1 4
E 0 9 3 6 1 4 5 2 7 8

Problem 7. Consider the following edge-weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.

![Graph Image]

a. What is the last edge that is added to the minimum spanning tree (MST) by Kruskal’s algorithm?

A 10
B 16
C 14
D 12
E 8

b. What is the weight of the MST?

A 45
B 48
C 36
D 50
E 56

Problem 8. Suppose that after running Dijkstra’s algorithm on an edge-weighted digraph, starting from vertex 0, the values in the `distTo` and `edgeTo` arrays are as shown below.

Initials:
a. What is the shortest path to vertex 3?

A 0 → 4 → 6 → 9 → 1 → 3
B 0 → 4 → 8 → 1 → 3
C 0 → 10 → 1 → 7 → 3
D 0 → 4 → 6 → 9 → 6 → 3
E 0 → 4 → 6 → 9 → 3

b. What is the weight on the edge 6 → 9?

A 8
B 2
C 6
D 4
E 10
Answers

Problem 1. C, B
Problem 2. C, A, C, A, D
Problem 3. E, B, A, C
Problem 4. A, B, E, B, A
Problem 5. D, A
Problem 6. C, D, A
Problem 7. C, A
Problem 8. E, D