Data Structures and Algorithms in Java

Searching: Hash Tables

Outline

Hashing

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- Computing the hash function
- Equality test: method for checking whether two keys are equal
- Collision resolution: algorithm and data structure to handle two keys that hash to the same array index

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Classic space-time tradeoff

- No space limitation: trivial hash function with key as index
- No time limitation: trivial collision resolution with sequential search
- Space and time limitations: hashing (the real world)

Hashing

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Practical challenge: need different approach for each type of key

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Java's hash code conventions

- All Java classes inherit a method `hashCode()`, which returns a 32-bit `int`
- Requirement: if `x.equals(y)`, then `x.hashCode() == y.hashCode()`
- Highly desirable: if `!x.equals(y)`, then `x.hashCode() != y.hashCode()`
- Default implementation: return memory address of `x`
- Legal (but poor) implementation: always return 17
- Customized implementations: `Integer`, `Double`, `String`, `File`, `URL`, `Date`, ...
- User-defined types: users are on their own

Hashing

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Java library implementations

```java
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        long bits = doubleToLongBits(value);
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```java
public final class String {
    private int hash = 0;
    private final char[] s;

    public int hashCode() {
        if (hash != 0) { return hash; }
        for (int i = 0; i < length(); i++) { hash = s[i] + (31 * hash); }
        return hash;
    }
}
```

Hashing

Hashing

Implementing hash code for user-defined types

```
public final class Transaction implements Comparable<Transaction> {
    private final String who;
    private final Date when;
    private final double amount;

    public int hashCode() {
        int hash = 17;
        hash = 31 * hash + who.hashCode();
        hash = 31 * hash + when.hashCode();
        hash = 31 * hash + ((Double) amount).hashCode();
        return hash;
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Hash code design

- Combine each significant field using the $31x + y$ rule
- If field is a primitive type, use wrapper type `hashCode()`
- If field is `null`, return 0
- If field is a reference type, use `hashCode()`
- If field is an array, apply to each entry

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Modular hashing

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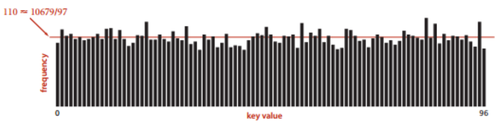
Example (hash value frequencies for words in Tale of Two Cities; 10,679 keys; $m = 97$)



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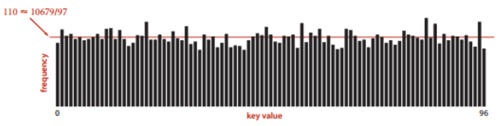


Collision: two distinct keys hash to the same index

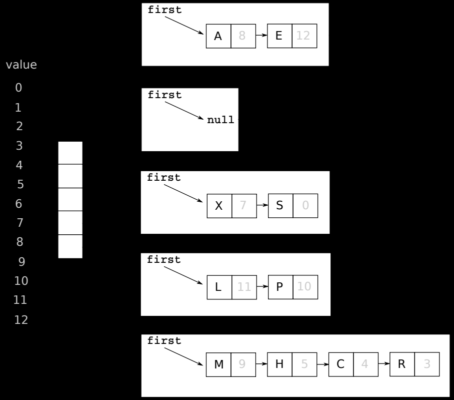
- Can't avoid collisions unless you have a ridiculous amount of memory
- Collisions are evenly distributed
- Challenge: deal with collisions efficiently

Separate-Chaining Symbol Table

Separate-Chaining Symbol Table

Use an array of $m < n$ linked lists

- Hash: map key to integer $i \in [0, m-1]$
- Insert: put at front of ith chain (if not already there)
- Search: need to search only the ith chain



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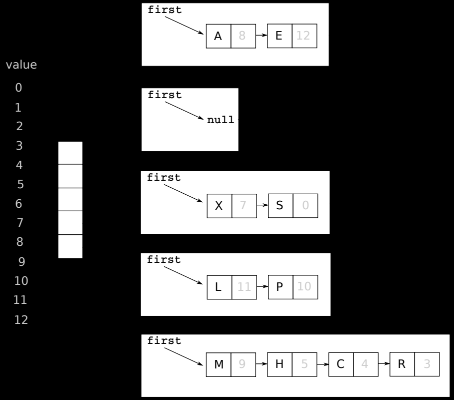

The ratio n/m is called the load factor and is denoted by α, and is interpreted as the average number of keys per list

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The cost of search, insert, and delete, under the uniform hashing assumption, is constant (between 3 and 5)

Separate-Chaining Symbol Table

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</> SeparateChainingHashST.java

```java
package dsa;

import stdlib.StdIn;
import stdlib.StdOut;

public class SeparateChainingHashST<Key, Value> implements BasicST<Key, Value> {
    private LinearSearchST<Key, Value>[] st;
    private int m;
    private int n;

    public SeparateChainingHashST() {
        this(4);
    }

    public SeparateChainingHashST(int m) {
        this.m = m;
        st = (LinearSearchST<Key, Value>[]) new LinearSearchST[m];
        for (int i = 0; i < m; i++) {
            st[i] = new LinearSearchST<Key, Value>();
        }
    }

    public boolean isEmpty() {
        return size() == 0;
    }

    public int size() {
        return n;
    }

    public void put(Key key, Value value) {
        if (key == null) {
            throw new IllegalArgumentException("key is null");
        }
        if (value == null) {
```

Separate-Chaining Symbol Table

SeparateChainingHashST.java

```java
            throw new IllegalArgumentException("value is null");
        }
        if (n >= 10 * m) {
            resize(2 * m);
        }
        int i = hash(key);
        if (!st[i].contains(key)) {
            n++;
        }
        st[i].put(key, value);
    }

    public Value get(Key key) {
        if (key == null) {
            throw new IllegalArgumentException("key is null");
        }
        int i = hash(key);
        return st[i].get(key);
    }

    public boolean contains(Key key) {
        if (key == null) {
            throw new IllegalArgumentException("key is null");
        }
        return get(key) != null;
    }

    public void delete(Key key) {
        if (key == null) {
            throw new IllegalArgumentException("key is null");
        }
        int i = hash(key);
        if (st[i].contains(key)) {
            n--;
        }
```

Separate-Chaining Symbol Table

SeparateChainingHashST.java

```java
        st[i].delete(key);
        if (m > 4 && n <= 2 * m) {
            resize(m / 2);
        }
    }

    public Iterable<Key> keys() {
        LinkedQueue<Key> queue = new LinkedQueue<Key>();
        for (LinearSearchST<Key, Value> chain : st) {
            for (Key key : chain.keys()) {
                queue.enqueue(key);
            }
        }
        return queue;
    }

    private int hash(Key key) {
        return (key.hashCode() & 0x7fffffff) % m;
    }

    private void resize(int chains) {
        SeparateChainingHashST<Key, Value> temp = new SeparateChainingHashST<Key, Value>(chains);
        for (LinearSearchST<Key, Value> chain : st) {
            for (Key key : chain.keys()) {
                temp.put(key, chain.get(key));
            }
        }
        this.m = temp.m;
        this.n = temp.n;
        this.st = temp.st;
    }

    public static void main(String[] args) {
        SeparateChainingHashST<String, Integer> st = new SeparateChainingHashST<String, Integer>();
        for (int i = 0; !StdIn.isEmpty(); i++) {
```

Separate-Chaining Symbol Table

SeparateChainingHashST.java

```java
            String key = StdIn.readString();
            st.put(key, i);
        }
        for (String s : st.keys()) {
            StdOut.println(s + " " + st.get(s));
        }
    }
}
```