Data Structures and Algorithms in Java

Sorting: Merge Sort

Outline

Merging

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Merge sort is based on an operation called merging: combining two ordered arrays to make one larger ordered array

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Example

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E

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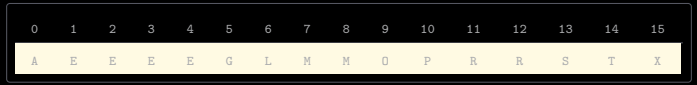

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A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Merging

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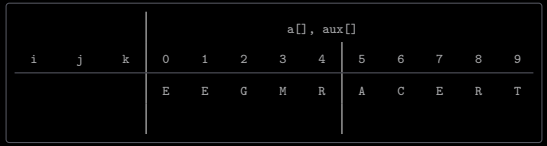


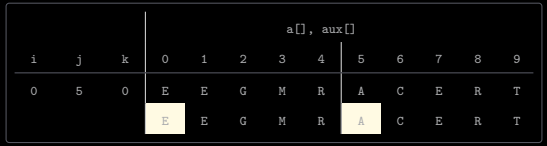
Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
			E	E	G	M	R	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
0	5		E	E	G	M	R	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

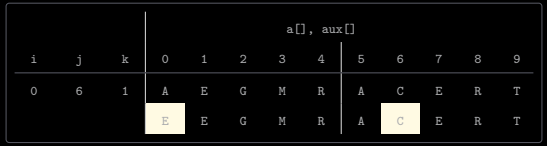
Merging



Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
0	6	1	A	E	G	M	R	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging



Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
0	7	2	A	C	G	M	R	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

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i	j	k	0	1	2	3	4	5	6	7	8	9
0	7	2	A	C	G	M	R	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

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1	7	3	A	C	E	M	R	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

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i	j	k	0	1	2	3	4	5	6	7	8	9
1	7	3	A	C	E	M	R	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
2	7	4	A	C	E	E	R	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
2	7	4	A	C	E	E	R	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

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i	j	k	0	1	2	3	4	5	6	7	8	9
2	8	5	A	C	E	E	E	A	C	E	R	T
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Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
2	8	5	A	C	E	E	E	A	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
3	8	6	A	C	E	E	E	G	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
3	8	6	A	C	E	E	E	G	C	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
4	8	7	A	C	E	E	E	G	M	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
4	8	7	A	C	E	E	E	G	M	E	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
5	8	8	A	C	E	E	E	G	M	R	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
5	8	8	A	C	E	E	E	G	M	R	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

			a[], aux[]									
i	j	k	0	1	2	3	4	5	6	7	8	9
5	9	9	A	C	E	E	E	G	M	R	R	T
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Merging

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5	9	9	A	C	E	E	E	G	M	R	R	T
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Merging

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i	j	k	0	1	2	3	4	5	6	7	8	9
5	10	10	A	C	E	E	E	G	M	R	R	T
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Merging

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i	j	k	0	1	2	3	4	5	6	7	8	9
			A	C	E	E	E	G	M	R	R	T
			E	E	G	M	R	A	C	E	R	T

Merging

Merging

</> Merge.java

```
public class Merge {
    private static void merge(Comparable[] a, Comparable[] aux, int lo, int mid, int hi) {
        for (int k = lo; k <= hi; k++) {
            aux[k] = a[k];
        }
        int i = lo, j = mid + 1;
        for (int k = lo; k <= hi; k++) {
            if (i > mid) {
                a[k] = aux[j++];
            } else if (j > hi) {
                a[k] = aux[i++];
            } else if (less(aux[j], aux[i])) {
                a[k] = aux[j++];
            } else {
                a[k] = aux[i++];
            }
        }
    }

    private static void merge(Object[] a, Object[] aux, int lo, int mid, int hi, Comparator c) {
        for (int k = lo; k <= hi; k++) {
            aux[k] = a[k];
        }
        int i = lo, j = mid + 1;
        for (int k = lo; k <= hi; k++) {
            if (i > mid) {
                a[k] = aux[j++];
            } else if (j > hi) {
                a[k] = aux[i++];
            } else if (less(aux[j], aux[i], c)) {
                a[k] = aux[j++];
            } else {
                a[k] = aux[i++];
            }
        }
    }
}
```

Sorting

Sorting

		a[]														
lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E

Sorting

		a[]														
lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	15	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E

Sorting

		a[]														
lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	7	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E

Sorting

		a[]														
lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	3	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E

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0	1	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E

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		a[]														
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0	1	E	M	R	G	E	S	O	R	T	E	X	A	M	P	L	E

Sorting

		a[]														
lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	3	E	M	R	G	E	S	O	R	T	E	X	A	M	P	L	E

Sorting

		a[]														
lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
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0	3	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E

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		a[]														
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4	7	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E

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lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	5	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E

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4	7	E	G	M	R	E	O	R	S	T	E	X	A	M	P	L	E

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0	7	E	G	M	R	E	O	R	S	T	E	X	A	M	P	L	E

Sorting

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0	7	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E

Sorting

		a[]														
lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
8	15	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E

Sorting

		a[]														
lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
8	11	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E

Sorting

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8	9	E	E	G	M	O	R	R	S	E	T	X	A	M	P	L	E

Sorting

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lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10	11	E	E	G	M	O	R	R	S	E	T	X	A	M	P	L	E

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8	11	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E

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12	15	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E

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12	13	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E

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14	15	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E

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lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
14	15	E	E	G	M	O	R	R	S	A	E	T	X	M	P	E	L

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lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
12	15	E	E	G	M	O	R	R	S	A	E	T	X	M	P	E	L

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lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	15	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X

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lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	15	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

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		a[]														
lo	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	15	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Sorting

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</> Merge.java

```java
public class Merge {
    public static void sort(Comparable[] a) {
        Comparable[] aux = new Comparable[a.length];
        sort(a, aux, 0, a.length - 1);
    }

    public static void sort(Object[] a, Comparator c) {
        Object[] aux = new Object[a.length];
        sort(a, aux, 0, a.length - 1, c);
    }

    private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi) {
        if (hi <= lo) {
            return;
        }
        int mid = lo + (hi - lo) / 2;
        sort(a, aux, lo, mid);
        sort(a, aux, mid + 1, hi);
        merge(a, aux, lo, mid, hi);
    }

    private static void sort(Object[] a, Object[] aux, int lo, int hi, Comparator c) {
        if (hi <= lo) {
            return;
        }
        int mid = lo + (hi - lo) / 2;
        sort(a, aux, lo, mid, c);
        sort(a, aux, mid + 1, hi, c);
        merge(a, aux, lo, mid, hi, c);
    }
}
```

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    public static void sort(Comparable[] a) {
        Comparable[] aux = new Comparable[a.length];
        sort(a, aux, 0, a.length - 1);
    }

    public static void sort(Object[] a, Comparator c) {
        Object[] aux = new Object[a.length];
        sort(a, aux, 0, a.length - 1, c);
    }

    private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi) {
        if (hi <= lo) {
            return;
        }
        int mid = lo + (hi - lo) / 2;
        sort(a, aux, lo, mid);
        sort(a, aux, mid + 1, hi);
        merge(a, aux, lo, mid, hi);
    }

    private static void sort(Object[] a, Object[] aux, int lo, int hi, Comparator c) {
        if (hi <= lo) {
            return;
        }
        int mid = lo + (hi - lo) / 2;
        sort(a, aux, lo, mid, c);
        sort(a, aux, mid + 1, hi, c);
        merge(a, aux, lo, mid, hi, c);
    }
}
```

$T(n) = n \log n$ and $S(n) = n$