Data Structures and Algorithms in Java

Sorting: Quick Sort

Outline

1. Partitioning
2. Sorting
3. An Algorithmic Improvement

Partitioning

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Quick sort works by partitioning the input array, about a pivot, into two subarrays, and then sorting the subarrays independently

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Example

a[]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Q	U	I	C	K	S	O	R	T	E	X	A	M	P	L	E

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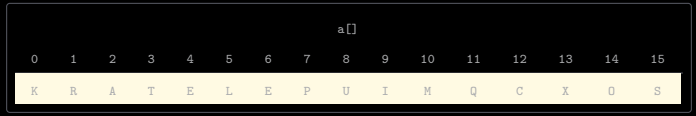

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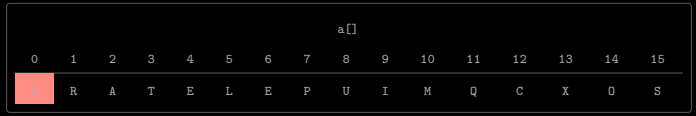

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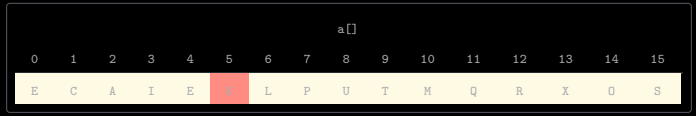

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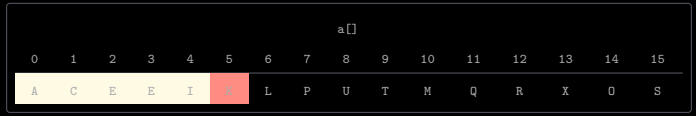

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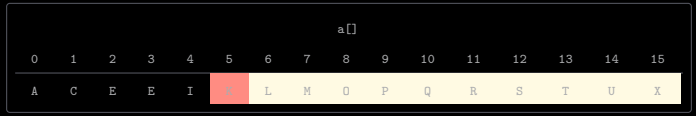

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- No entry in `a[lo]` through `a[j - 1]` is greater than `a[j]`
- No entry in `a[j + 1]` through `a[hi]` is less than `a[j]`

Partitioning

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i	j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S

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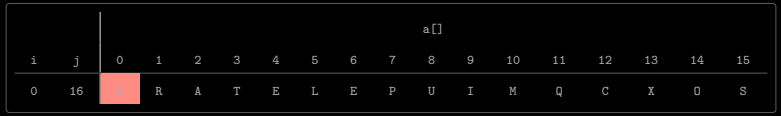

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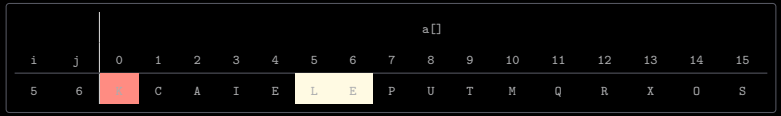

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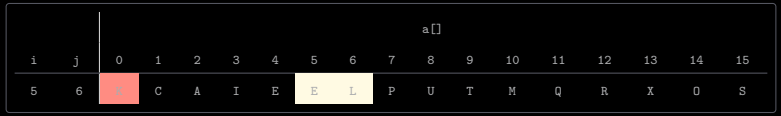

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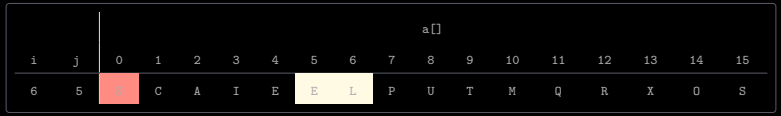

Partitioning

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	5	E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S

Partitioning

Partitioning

</> Quick.java

```
public class Quick {
    private static int partition(Comparable[] a, int lo, int hi) {
        int i = lo;
        int j = hi + 1;
        Comparable v = a[lo];
        while (true) {
            while (less(a[++i], v)) {
                if (i == hi) {
                    break;
                }
            }
            while (less(v, a[--j])) {
                if (j == lo) {
                    break;
                }
            }
            if (i >= j) {
                break;
            }
            exchange(a, i, j);
        }
        exchange(a, lo, j);
        return j;
    }

    private static int partition(Object[] a, int lo, int hi, Comparator c) {
        int i = lo;
        int j = hi + 1;
        Object v = a[lo];
        while (true) {
            while (less(a[++i], v, c)) {
                if (i == hi) {
                    break;
                }
            }
```

Partitioning

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```
            while (less(v, a[--j], c)) {
                if (j == lo) {
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                }
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            exchange(a, i, j);
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lo	j	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
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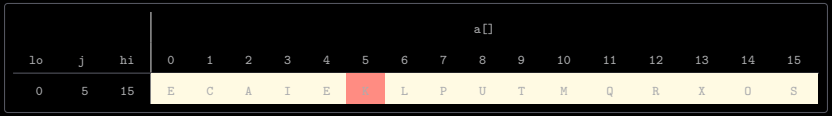

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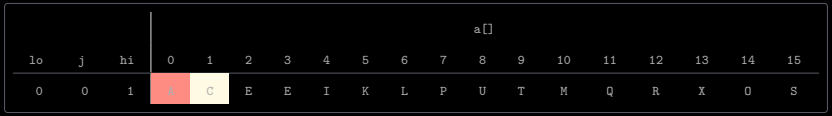

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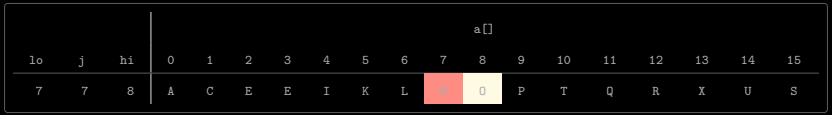

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        StdRandom.shuffle(a);
        sort(a, 0, a.length - 1);
    }

    public static void sort(Object[] a, Comparator c) {
        StdRandom.shuffle(a);
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    }

    private static void sort(Comparable[] a, int lo, int hi) {
        if (hi <= lo) {
            return;
        }
        int j = partition(a, lo, hi);
        sort(a, lo, j - 1);
        sort(a, j + 1, hi);
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    private static void sort(Object[] a, int lo, int hi, Comparator c) {
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$T(n) = n \log n$

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When the input array has a large number of duplicates, 3-way quick sort can significantly improve performance

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3-way quick sort partitions the array into three parts, one each for keys smaller than, equal to, and larger than the pivot

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lt	i	gt	0	1	2	3	4	5	6	7	8	9	10	11
			B	R	B	R	W	W	W	R	R	W	R	B

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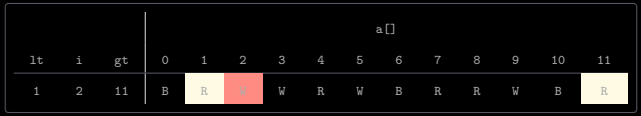

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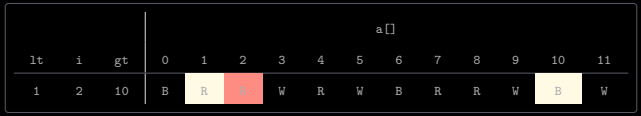

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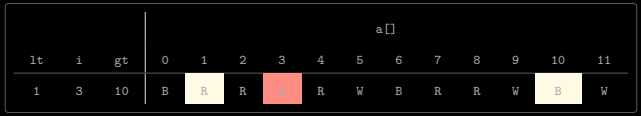

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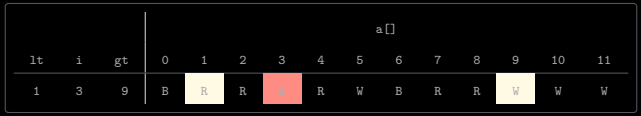

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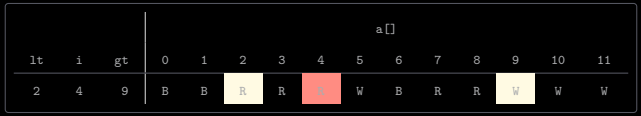

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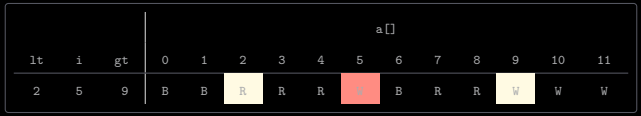

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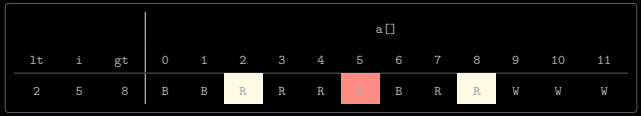

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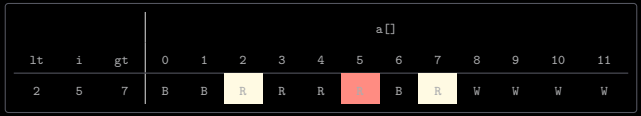

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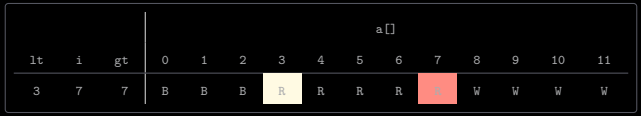

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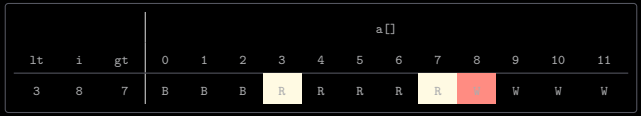

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lt	i	gt	0	1	2	3	4	5	6	7	8	9	10	11
3	8	7	B	B	B	R	R	R	R	R	W	W	W	W

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</> Quick3way.java

```
public class Quick3way {
    public static void sort(Comparable[] a) {
        StdRandom.shuffle(a);
        sort(a, 0, a.length - 1);
    }

    public static void sort(Object[] a, Comparator c) {
        StdRandom.shuffle(a);
        sort(a, 0, a.length - 1, c);
    }

    private static void sort(Comparable[] a, int lo, int hi) {
        if (hi <= lo) {
            return;
        }
        int lt = lo, gt = hi;
        Comparable v = a[lo];
        int i = lo + 1;
        while (i <= gt) {
            int cmp = a[i].compareTo(v);
            if (cmp < 0) {
                exchange(a, lt++, i++);
            } else if (cmp > 0) {
                exchange(a, i, gt--);
            } else {
                i++;
            }
        }
        sort(a, lo, lt - 1);
        sort(a, gt + 1, hi);
    }

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An Algorithmic Improvement

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```
        }
        int lt = lo, gt = hi;
        Object v = a[lo];
        int i = lo + 1;
        while (i <= gt) {
            int cmp = c.compare(a[i], v);
            if (cmp < 0) {
                exchange(a, lt++, i++);
            } else if (cmp > 0) {
                exchange(a, i, gt--);
            } else {
                i++;
            }
        }
        sort(a, lo, lt - 1, c);
        sort(a, gt + 1, hi, c);
    }
}
```

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Given an array of n keys with k of them distinct, let f_i be the frequency of the ith key and $p_i = f_i/n$ the probability that the ith key is found when the array is sampled

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The Shannon entropy of the keys is defined as

$$\begin{aligned} H &= -(p_1 \lg p_1 + p_2 \lg p_2 + \cdots + p_k \lg p_k) \\ &= -\sum_{i=1}^{k} p_i \lg p_i \end{aligned}$$

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Note that $0 \leq H \leq \ln n$

Running time for 3-way quick sort in terms of Shannon entropy is $T(n) = nH$