Exercise 1. Consider a language over the alphabet $\{a, b\}$ that consists of strings with zero or more as or bs.

a. Provide a regular expression for the language.

b. Use Thompson's algorithm to derive an equivalent non-deterministic finite state automaton (NFA).

c. Use powerset construction algorithm to derive an equivalent deterministic finite state automaton (DFA).

d. Use partition refinement algorithm to derive an equivalent minimal DFA.

Exercise 2. Consider a language over the alphabet $\{a, b\}$ that consists of strings ending with an ab.

a. Provide a regular expression for the language.

b. Use Thompson's algorithm to derive an equivalent non-deterministic finite state automaton (NFA).

c. Use powerset construction algorithm to derive an equivalent deterministic finite state automaton (DFA).

d. Use partition refinement algorithm to derive an equivalent minimal DFA.

Exercise 3. Consider a language over the alphabet $\{a, b\}$ that consists of strings starting with an a and ending with a b.

a. Provide a regular expression for the language.

b. Use Thompson's algorithm to derive an equivalent non-deterministic finite state automaton (NFA).

c. Use powerset construction algorithm to derive an equivalent deterministic finite state automaton (DFA).

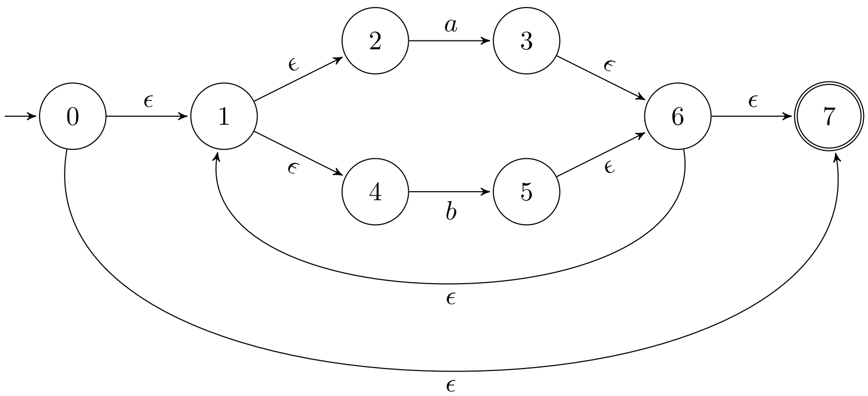
d. Use partition refinement algorithm to derive an equivalent minimal DFA.

Solutions

Solution 1.

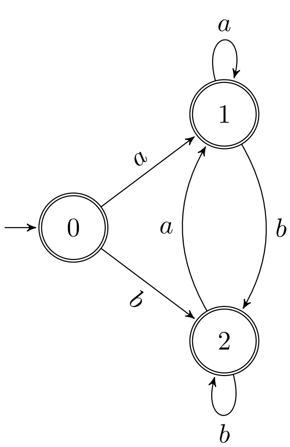
a. $(a|b)*$

b.

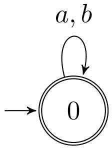


c.

r	a	$m(r,a)$
ϵ-closure(0) $= \{0,1,2,4,7\} = 0$ (final)	a	ϵ-closure(3) $= \{1,2,3,4,6,7\} = 1$ (final)
0	b	ϵ-closure(5) $= \{1,2,5,4,6,7\} = 2$ (final)
1	a	ϵ-closure(3) $= 1$
1	b	ϵ-closure(5) $= 2$
2	a	ϵ-closure(3) $= 1$
2	b	ϵ-closure(5) $= 2$



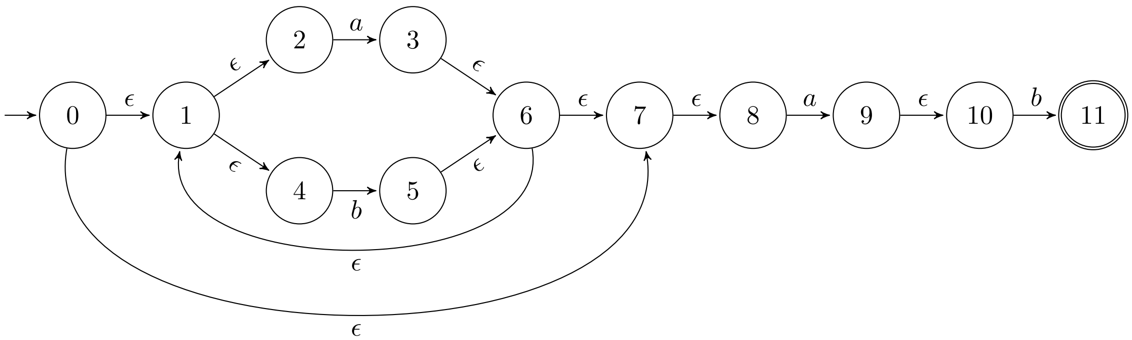
d. The initial (and final) partition is $\mathcal{P} = \{\{0,1,2\}\}$. Labeling the subset $\{0,1,2\}$ as 0, we have the following minimal DFA:



Solution 2.

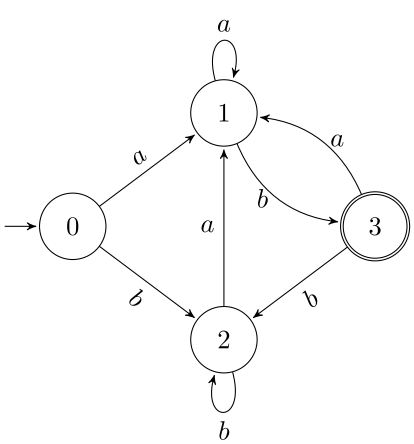
a. $(a|b)*ab$

b.

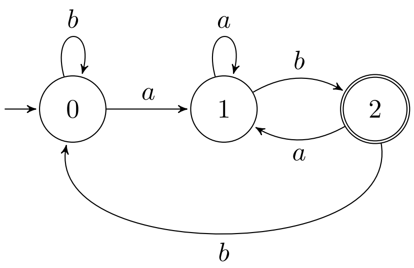


c.

$\boldsymbol{r}$	$\boldsymbol{a}$	$\boldsymbol{m(r, a)}$
ϵ-closure(0) $= \{0, 1, 2, 4, 7, 8\} = 0$	a	ϵ-closure(3) $\cup$ ϵ-closure(9) $= \{1, 2, 3, 4, 6, 7, 8, 9, 10\} = 1$
0	b	ϵ-closure(5) $= \{1, 2, 4, 5, 6, 7, 8\} = 2$
1	a	ϵ-closure(3) $\cup$ ϵ-closure(9) $= 1$
1	b	ϵ-closure(5) $\cup$ ϵ-closure(11) $= \{1, 2, 4, 5, 6, 7, 8, 11\} = 3$ (final)
2	a	ϵ-closure(3) $\cup$ ϵ-closure(9) $= 1$
2	b	ϵ-closure(5) $= 2$
3	a	ϵ-closure(3) $\cup$ ϵ-closure(9) $= 1$
3	b	ϵ-closure(5) $= 2$



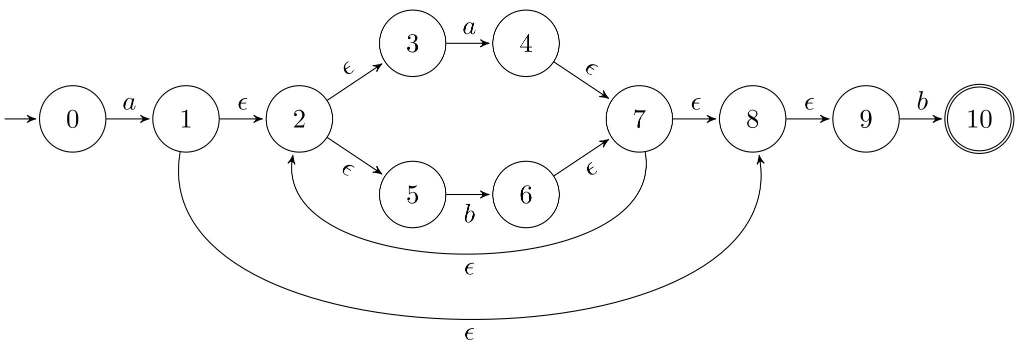
d. The initial partition is $\mathcal{P} = \{\{0, 1, 2\}, \{3\}\}$. The subset $\{0, 1, 2\}$ splits on b into subsets $\{0, 2\}$ and $\{1\}$. The final partition is $\mathcal{P} = \{\{0, 2\}, \{1\}, \{3\}\}$. Labeling the subsets $\{0, 1, 2\}$ as 0, 1, and 2, we have the following minimal DFA:



Solution 3.

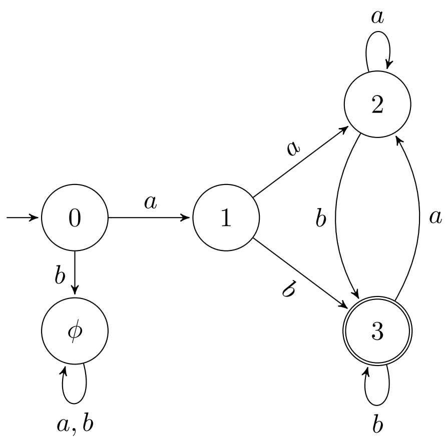
a. $a(a|b)*b$

b.



c.

r	a	$m(r, a)$
ϵ-closure$(0) = \{0\} = 0$	a	ϵ-closure$(1) = \{1, 2, 3, 5, 8, 9\} = 1$
0	b	ϕ (dead state)
1	a	ϵ-closure$(4) = \{2, 3, 4, 5, 7, 8, 9\} = 2$
1	b	ϵ-closure$(6) \cup \epsilon$-closure$(10) = \{2, 3, 5, 6, 7, 8, 9, 10\} = 3$ (final)
2	a	ϵ-closure$(4) = 2$
2	b	ϵ-closure$(6) \cup \epsilon$-closure$(10) = 3$
3	a	ϵ-closure$(4) = 2$
3	b	ϵ-closure$(6) \cup \epsilon$-closure$(10) = 3$
ϕ	a	ϕ
ϕ	b	ϕ



d. The initial partition is $\mathcal{P} = \{\{0, 1, 2, \phi\}, \{3\}\}$. The subset $\{0, 1, 2, \phi\}$ splits on b into subsets $\{0, \phi\}$ and $\{1, 2\}$. The subset $\{0, \phi\}$ further splits on a into subsets $\{0\}$ and ϕ. The final partition is $\mathcal{P} = \{\{0\}, \{1, 2\}, \{3\}, \phi\}$. Labeling the subsets $0, 1, 2$, and ϕ, we have the following minimal DFA: