

UMass Boston Computer Science  
**CS450 High Level Languages**

# Implementing Lambda Functions

Thursday, April 16, 2026



# *Logistics*

- HW 9 in
  - ~~due: Tues 4/14 11am EST~~
  - ~~due: Thur 4/16 11am EST~~
- HW 10 out
  - ~~due: Tues 4/21 11am EST~~
  - due: Thur 4/23 11am EST



Last Time

# Function Application in CS450 Lang

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(fnapp ,Program . ,List<Program>)
```

function

arguments

“rest” arg (like cons)

Specifies arbitrary number of args

# Function Application in CS450 Lang: Examples

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(fnapp ,Program . ,List<Program>)
```

function

arguments

```
(fnapp + 1 2)
```

Programmers should not need to write the explicit “fnapp”!

# Function Application in CS450 Lang: Examples


```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(fnapp ,Program . ,List<Program>)
```

```
(fnapp + 1 2)
```

# Function Application in CS450 Lang: Examples

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(,Program . ,List<Program>)
```

“rest” arg (like cons)



```
(+ 1 2)
```

# Function Application in CS450 Lang: Examples

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(,Program . ,List<Program>)  
;; - (cons Program List<Program>)
```

(+ 1 2)

Must be careful when parsing this!

Function call case (must be last, why?)

Can now go back to (simpler?) cons notation

(bind 1 2)

should not be parsed  
as function call!

```

(define/contract (parse p)
  (-> Program? AST?)
  (match p
    ...
    [ `(bind [,(and (? symbol?) x) ,e] ,bod) ... ]
    [ `(bind . ,_) Raise bind parse error here
      (raise-syntax-error 'parse "invalid bind syntax"
        #:exn exn:fail:syntax:cs450) ]
    [_ (raise-syntax-error
        'parse "not a valid CS450 Lang program" p
        #:exn exn:fail:syntax:cs450))]))

```

(bind 1 2)

should not be parsed  
as function call!

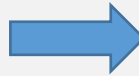
(another case below  
should not match this  
error case successfully)



# Function Application in CS450 Lang

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - (cons Program List<Program>)
```

parse



```
;; An AST is one of:  
;; ...  
;; - (mk-appl AST List<AST>)  
;; ...  
(struct appl [fn args])
```

# “Running” Function Calls

```
;; run: AST -> Result
```

```
(define (run p)
```

```
(define (run/e p env)  
  (match p
```

```
    ...  
    [(appl fn args) (apply  
                      (run/e fn env)  
                      (map (curryr run/e env) args))]  
    ...
```

```
  ))  
(run/e p INIT-ENV))
```

```
;; An AST is one of:  
;; ...  
;; - (mk-appl AST List<AST>)  
;; ...  
(struct appl [fn args])
```

# “Running” Function Calls

```
;; run: AST -> Result
```

```
(define (run p)
```

```
  (define (run/e p env)
```

```
    (match p
```

```
      TEMPLATE: extract pieces of compound data
```

```
      ...
```

```
      [(appl fn args) (apply  
                        (run/e fn env)  
                        (map (curryr run/e env) args))])
```

```
      ...
```

```
    ))
```

```
(run/e p INIT-ENV))
```

```
;; An AST is one of:
```

```
;; ...
```

```
;; - (mk-appl AST List<AST>)
```

```
;; ...
```

```
(struct appl [fn args])
```



# “Running” Function Calls

```
;; run: AST -> Result
```

```
(define (run p)
```

```
  (define (run/e p env)
```

```
    (match p
```

```
      ...
```

```
      [(appl fn args)
```

```
        (run/e fn env)
```

```
        (map (curry ??? run/e env) args)))]
```

```
      ...
```

```
    ))
```

```
  (run/e p INIT-ENV))
```

```
;; An AST is one of:  
;; ...  
;; - (mk-appl AST List<AST>)  
;; ...  
(struct appl [fn args])
```

TEMPLATE: recursive calls

List-processing function

# “Running” Function Calls

How do we actually run the function?

;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - (Racket) Function

```
(define (run p)
```

```
(define (run/e p env)
```

```
(match p
```

```
...
```

```
[(appl fn args) (apply
```

Runs a Racket function

```
(run/e fn env)
```

function

```
(map (curryr run/e env) args))
```

List of args

```
...
```

Does this work?

```
))
```

```
(run/e p INIT-ENV))
```

# “Running” Non-Functions

```
(define (run p)
```

```
(define (run/e p env)
  (match p
```

```
...
```

```
[(appl fn args) (apply
                  (run/e fn env)
                  (map (curryr run/e env) args))]
```

```
...
```

```
))
(run/e p INIT-ENV))
```

```
;; A Result is one of:
;; - Number
;; - UNDEFINED-ERROR
;; - (Racket) Function
```

```
Example: (eval450 '(10 10)) ; apply non-fn
```

# “Running” Non-Functions

```
(define (run p)
```

```
(define (run/e p env)  
  (match p
```

```
    ...  
    [(appl fn args) (450apply  
                      (run/e fn env)  
                      (map (curryr run/e env) args))]  
    ...  
  ))
```

```
(run/e p INIT-ENV))
```

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

Example: (eval450 '(10 10)) ; apply non-fn

# Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)  
  (cond  
    [(number? fn) ... ]  
    [(UNDEFINED-ERROR? fn) ... ]  
    [(NON-FUNCTION-ERROR? fn) ... ]  
    [(procedure? fn) ... ]))
```

TEMPLATE?

# Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)  
  (cond  
    [(number? fn) NON-FUNCTION-ERROR]  
    [(UNDEFINED-ERROR? fn) ... ]  
    [(NON-FUNCTION-ERROR? fn) ... ]  
    [(procedure? fn) ... ]))
```

# Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)  
  (cond  
    [(number? fn) NON-FUNCTION-ERROR]  
    [(UNDEFINED-ERROR? fn) ...]  
    [(NON-FUNCTION-ERROR? fn) ...]  
    [(procedure? fn) (apply fn args)])))
```

Now this works

# Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)  
  (cond  
    [(number? fn) NON-FUNCTION-ERROR]  
    [(UNDEFINED-ERROR? fn) ... ]  
    [(NON-FUNCTION-ERROR? fn) NON-FUNCTION-ERROR]  
    [(procedure? fn) (apply fn args)]))
```

# Function application for CS450 Lang

```
;; A Result is one of:  
;; - Number  
;; - UNDEFINED-ERROR  
;; - NON-FUNCTION-ERROR  
;; - (Racket) Function
```

```
;; 450apply : Result Listof<Result> -> Result
```

```
(define (450apply fn args)  
  (cond  
    → (number? fn) NON-FUNCTION-ERROR ]  
    [(UNDEFINED-ERROR? fn) UNDEFINED-ERROR ]  
    → (NON-FUNCTION-ERROR? fn) NON-FUNCTION-ERROR ]  
    [(procedure? fn) (apply fn args)]))
```

UNDEFINED should have precedence over NON-FN-ERR

Combine cases?

# Function application for CS450 Lang

```
;; A Result is one of:
;; - Number
;; - UNDEFINED-ERROR
;; - NON-FUNCTION-ERROR
;; - (Racket) Function
```

Add ARITY-ERROR ???

```
;; 450apply : Result Listof<Result> -> Result
```

For now, we only use variable-arity functions

```
(define (450apply fn args)
  (cond
    [(UNDEFINED-ERROR? fn) UNDEFINED-ERROR]
    [(procedure? fn) (apply fn args)]
    [else NON-FUNCTION-ERROR]))
```

```
(define INIT-ENV
  `((+ ,450+)
    (× ,450*)))
```

These should have "variable arity" (like Racket +)

Check correct number of arguments???

Combine cases

# Interlude: Variable-arity functions in Racket

Programmer should not be constructing a list

```
;; 450+: List<Result> -> Result ???
```

```
;; 450+: Result ... -> Result
```

```
(define/contract (450+ . args)
  (-> Result? ... Result? )
  ... )
```

In the function body, **args** is a list of arguments

This should now have  
“**variable arity**”  
(like Racket +)

(compare with JS “variadic” args)

```
function sum(...theArgs) {
  let total = 0;
  for (const arg of theArgs) {
    total += arg;
  }
  return total;
}
```

# Function Application in CS450 Lang

```
;; A Program is one of:  
;; - Atom  
;; - Variable  
;; - `(bind [,Var ,Program] ,Program)  
;; - (cons Program List<Program>)
```

Function call case (must be last)

What if a user wants to define their own function!

Next Feature: Lambdas?

# Function Application in CS450 Lang

What functions can be called?

(+ 1 2)

1. (Racket) functions in initial environment

(??? 1 2)

2. user-defined (“lambda”) functions?

# “Lambdas” in CS450 Lang

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - (cons Program List<Program>)
```

# “Lambdas” in CS450 Lang

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(lm ,List<Var> ,Program)  
;; - (cons Program List<Program>)
```

# CS450 Lang “Lambda” examples

CS450LANG

`(lm (x y) (+ x y))`

;; A Program is one of:

;; - Atom

;; - Variable (Var)

;; - `(bind [ ,Var ,Program] ,Program)

;; - `(lm ,List<Var> ,Program)

;; - (cons Program List<Program>)

Equivalent to ...

RACKET

`(lambda (x y) (+ x y))`

`(lm (x) (lm (y) (+ x y))) ; “curried”`

`( (lm (x y) (+ x y)) 10 20 ) ; lm applied`

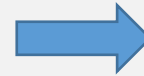
# Parsing “Lambda”

CS450LANG

```
(lm (x y) (+ x y))
```

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(lm ,List<Var> ,Program)  
;; - (cons Program List<Program>)
```

parse



```
;; An AST is one of:  
;; ...  
;; - (mk-lm-ast List<Symbol> AST)  
;; ...
```

Be careful when parsing, compare to RACKET `lambda`:

Welcome to [DrRacket](#), version 8.10 [65].

Language: racket, with test coverage [custom]; memory limit: 10M

```
> (lambda)
```

```
✘ Lambda: bad syntax in: (lambda)
```

```
> (lambda 1)
```

```
✘ Lambda: bad syntax in: (lambda 1)
```

```
> (lambda (1) 2)
```

```
✘ Lambda: not an identifier, identifier with default, or keyword in: 1
```

```
(struct lm-ast [params body])  
;; ...
```

# Parsing “Lambda”

CS450LANG

(lm (x y) (+ x y))

```
(define/contract (parse p)
  (-> Program? AST?)
  (match p
```

Pattern that matches  
correct lm syntax

```
...
[ `(lm ,(and (list (? symbol?) ...) args) ,bod) ... ]
```

```
[ `(,fn . ,args) ... ]
```

```
[_ (raise-syntax-error
    'parse "not a valid CS450 Lang program" p
    #:exn exn:fail:syntax:cs450))])
```

# Parsing “Lambda”

```
(define/contract (parse p)
  (-> Program? AST?)
  (match p
    ...
    [ `(lm ,(and (list (? symbol?) ...) args) ,bod) ... ]
    [ `(lm . ,_)
      (raise-syntax-error 'parse "invalid lm syntax" p
        #:exn exn:fail:syntax:cs450) ]
    [ `(,fn . ,args) ... ]
    [ _ (raise-syntax-error
          'parse "not a valid CS450 Lang program" p
          #:exn exn:fail:syntax:cs450) ] ]))
```

“Lambda” parse error case

User-defined exception

# CS450 Lang “Lambda” AST node

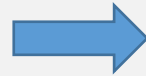
```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(lm ,List<Var> ,Program)  
;; - (cons Program List<Program>)
```

eval450



```
;; A Result is one of  
;; - ???
```

parse



```
;; An AST is one of:  
;; ...  
;; - (mk-lm-ast List<Symbol> AST)  
;; ...
```

```
(struct lm-ast [params body])  
;; ...
```

run



This represents code  
(that has not been run)!

# CS450 Lang “Lambda” full examples

```
(check-equal?  
  (eval450  
    '(bind [x 10]  
          ( (lm (y) (+ x y)) 20 )))  
  30 ) ; with bind
```

Can reference non-parameter bindings

```
(check-equal?  
  (eval450  
    '( (bind [x 10]  
        (lm (y) (+ x y))))  
    20 )  
  30 ) ; with bind (lm only)
```

Expression that evaluates to a function result

argument → 20

```
(check-equal?  
  (eval450  
    '( (lm (x y) (+ x y))  
      10 20 ) )  
  ? )
```

# In-class Coding 4/16: `lm` scope examples

```
(check-equal?  
  (eval450  
    '(bind [x 10]  
           ( (lm (y) (+ x y)) 20 )))  
    30 ) ; with bind
```

```
(check-equal?  
  (eval450  
    '( (bind [x 10]  
          (lm (y) (+ x y))))  
      20 ))  
    30 ) ; with bind (lm only)
```

Expression that evaluates to a function result

argument → 20

Come up with some of your own!

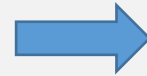
```
(check-equal?  
  (eval450  
    '( (lm (x y) (+ x y))  
      10 20 ))  
    30 )
```

Check your understanding in Racket! (using `lambda` and `let`)

# CS450 Lang “Lambda” AST node

```
;; A Program is one of:  
;; - Atom  
;; - Variable (Var)  
;; - `(bind [,Var ,Program] ,Program)  
;; - `(lm ,List<Var> ,Program)  
;; - (cons Program List<Program>)
```

parse



```
;; An AST is one of:  
;; ...  
;; - (mk-lm-ast List<Symbol> AST)  
;; ...  
>(struct lm-ast [params body])
```

Why can't we use a Racket `lambda`?

Because this **represents code!**

A Racket `lambda` is a “Result”, e.g., you can't “get” the parameters or the body code (it's not “transparent”)

# “Running” Functions?

```
;; run: AST -> Result
```

```
(define (run p)
```

TEMPLATE?

```
(define (run/e p env)
```

```
(match p
```

```
...
```

```
[(lm-ast params body) ??] ??] ??]
```

```
...
```

```
))
```

```
(run/e p INIT-ENV))
```

```
;; An AST is one of:
```

```
;; ...
```

```
;; - (mk-lm-ast List<Symbol> AST)
```

```
;; ...
```

```
(struct lm-ast [params body])
```

# “Running” Functions?

```
;; run: AST -> Result
```

```
(define (run p)
```

TEMPLATE

```
(define (run/e p env)
```

```
(match p
```

```
...
```

```
[(lm-ast params body) ?? params ?? (run/e body env) ??]
```

```
...
```

```
))
```

```
(run/e p INIT-ENV))
```

```
;; An AST is one of:
```

```
;; ...
```

```
;; - (mk-lm-ast List<Symbol> AST)
```

```
;; ...
```

```
(struct lm-ast [params body])
```

# “Running” Functions?

```
;; run: AST -> Result
```

```
(define (run p)
```

```
(define (run/e p env)  
  (match p
```

...

```
    [(lm-ast params body) ?? params ?? (run/e body env) ??]
```

What should be the “Result” of running an `lm` function?

```
    ))
```

```
(run/e p
```

Can we “convert” a 450lang “`lm`” AST into a Racket function???

```
;; An AST is one of:
```

```
;; ...
```

```
;; - (mk-lm-ast List<Symbol> AST)
```

```
;; ...
```

```
(struct lm-ast [params body])
```

```
;; A Result is one of:
```

```
;; - Number
```

```
;; - ErrorResult
```

```
;; - (Racket) Function ???
```

**We can't!!** (it's not “transparent”) (this is what makes FFIs and mixed lang progs complicated) So we need some other representation

e.g., can't call Python function from Java, etc

# “Running” Functions?

Can we “convert” this into a Racket function?

```
;; An AST is one of:  
;; ...  
;; - (mk-lm-ast List<Symbol> AST)  
;; ...  
(struct lm-ast [params body])
```

WAIT! Are **lm-result** and **lm-ast** the same?

```
;; A Result is one of:  
;; - Number  
;; - ErrorResult  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST ??)  
(struct lm-result [params body ??])
```

**We can't!!** need some other representation

# “Running” Functions? Full example

```
(bind [x 10]
      (lm (y) (+ x y)))
```

parse



???

```
(bind 'x (num 10)
      (lm-ast '(y)
               (appl (var '+)
                      (list (var 'x) (var 'y)))))
```

In Racket (with `lambda` and `let`)

Welcome to [DrRacket](#), version 8.10 [cs].

Language: racket, with test coverage [custom]; memory limit: 1024

```
> (define f
    (let ([x 10])
      (lambda (y) (+ x y))))
```

```
> x
x: undefined;
cannot reference an identifier before its definition
```

```
> (f 100)
```

```
110
```

run



```
result '(y)
  (var '+)
  (list (var 'x) (var 'y))
```

Where is the x???

```
(Racket) Function
mk-lm-res List<Symbol> AST ??)
t lm-result [params body ??])
```

# “Running” Functions? Full example

```
(bind [x 10]  
      (lm (y) (+ x y)))
```

parse



???

```
(bind 'x (num 10)  
      (lm-ast '(y)  
                (appl (var '+)  
                        (list (var 'x) (var 'y)))))
```

run



```
(lm-result '(y)  
           (appl (var '+)  
                 (list (var 'x) (var 'y))))
```

Where is the x???

lm-result must save the x!!

(how can we “remember” the x)

# “Running” Functions?

```
;; An AST is one of:  
;; ...  
;; - (mk-lm-ast List<Symbol> AST)  
;; ...  
(struct lm-ast [params body])
```

WAIT! Are **lm-result** and **lm-ast** the same?

```
;; A Result is one of:  
;; - Number  
;; - ErrorResult  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST ??)  
(struct lm-result [params body ??])
```

# “Running” Functions?

Quiz:

**Q:** What is the difference between **lm-ast** and **lm-result**?

;; An AST is one of:

;; ...

**A:** **lm-ast** is AST data

- represents code that a programmer writes (hasn't been run!);

**lm-result** is Result data

- represents result of running the program

(importantly contains **environment** for variables that are not fn parameters)

```
;; An lm function Result needs an extra environment
;; (for the non-argument variables used in the body!)
;; - ERRORRESULT
;; - (Racket) Function
;; - (mk-lm-res List<Symbol> AST Env)
(struct lm-result [params body env])
```

# “Running” Functions?

```
;; run: AST -> Result
```

```
(define (run p)
```

```
(define (run/e p env)  
  (match p
```

...

```
    [(lm-ast params body) ?? params ?? (run/e body env) ??]
```

What should be the “Result” of running a function?

```
  ))
```

```
(run/e p
```

Can we “convert” a 450lang “lm” AST into a Racket function???

**We can’t!!** need some other representation

```
;; An AST is one of:
```

```
;; ...
```

```
;; - (mk-lm-ast List<Symbol> AST)
```

```
;; ...
```

```
(struct lm-ast [params body])
```

```
;; A Result is one of:
```

```
;; - Number
```

```
;; - ErrorResult
```

```
;; - (Racket) Function ???
```

# “Running” Functions?

```
;; run: AST -> Result
```

```
(define (run p)
```

```
(define (run/e p env)
```

```
(match p
```

```
...
```

```
[(lm-ast params body) ?? params ?? (run/e body env) ??]
```

What should be the “Result” of running a function?

```
)
```

```
(run/e p INIT-ENV))
```

```
;; An AST is one of:
```

```
;; ...
```

```
;; - (mk-lm-ast List<Symbol> AST)
```

```
;; ...
```

```
(struct lm-ast [params body])
```

```
;; A Result is one of:
```

```
;; - Number
```

```
;; - ErrorResult
```

```
;; - (Racket) Function
```

```
;; - (mk-lm-res List<Symbol> AST Env)
```

```
(struct lm-result [params body env])
```

# Result of “Running” a Function

```
;; run: AST -> Result
```

```
(define (run p)
```

```
  (define (run/e p env)
```

```
    (match p
```

```
      ... body won't get “run” until the function is called
```

```
      [(lm-ast params body) (mk-lm-res params body env)]
```

```
      ...
```

```
    ))
```

```
(run/e p INIT-ENV))
```

Save the current env



Previously

# Running Function “Calls”: Revisited

How do we actually run the function?

```
; A Result is one of:  
; - Number  
; - ErrorResult  
; - (Racket) Function
```

```
(define (run p)
```

```
(define (run/e p env)
```

```
(match p
```

```
...
```

```
[(appl fn args) (apply  
                  (run/e fn env)  
                  (map (curryr run/e env) args))])
```

```
...
```

```
))  
(run/e p INIT-ENV))
```

Runs a Racket function

???

Does this work???

# Running Function “Calls”: Revisited

How do we actually run the function?

```
(define (run p)
```

```
(define (run/e p env)  
  (match p
```

```
    ...  
    [(appl fn args) (450apply  
                      (run/e fn env)  
                      (map (curryr run/e env) args))])  
    ...
```

(this doesn't “work” anymore!)

```
  ))  
(run/e p INIT-ENV))
```

```
; A Result is one of:  
;; - Number  
;; - ErrorResult  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST Env)  
(struct lm-result [params body env])
```

apply doesn't work for `lm-result`!!  
must manually implement “function call”

# CS450 Lang “Apply”

```
;; A Result is one of:  
;; - ...  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST Env)  
(struct lm-result [params body env])
```

```
;; 450apply : Result List<Result> -> Result  
(define (450apply fn args)  
  ...  
)
```

# CS450 Lang “Apply”

TEMPLATE

```
;; A Result is one of:  
;; - ...  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST Env)  
(struct lm-result [params body env])
```

```
;; 450apply : Result List<Result> -> Result  
(define (450apply fn args)  
  (match fn  
    ...  
    [(? procedure?) ...] ;; racket function  
    [(lm-result params body env) ...] ;; user-defined function  
    ... params ... body ... env)))
```

# CS450 Lang “Apply”

TEMPLATE: mutually referential data and template calls!

```
;; A Result is one of:  
;; - ...  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST Env)  
(struct lm-result [params body env])
```

```
;; 450apply : Result List<Result> -> Result  
(define (450apply fn args)  
  (match fn  
    ...  
    [(? procedure?) ... ] ;; racket function  
    [(lm-result params body env) ;; user-defined function  
     ... params ... (ast-fn body ... ) ... (env-fn env ... ) ... ]))
```

env-add : Env Var Result -> Env

# CS450 Lang “Apply”

```
;; A Result is one of:  
;; - ...  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST Env)  
(struct lm-result [params body env])
```

```
;; 450apply : Result List<Result> -> Result  
(define (450apply fn args)  
  (match fn  
    ...  
    [(? procedure?) ... ] ;; racket function  
    [(lm-result params body env) ;; user-defined function  
     ... (ast-fn body ... ) ... (env-add env ?? args params ?? ) ... ]))
```

Wait, these are lists

env-add : Env Var Result -> Env

# CS450 Lang “Apply”

(so this function should be `internal` in `run`)

```
;; A Result is one of:  
;; - ...  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST Env)  
(struct lm-result [params body env])
```

```
;; 450apply : Result List<Result> -> Result  
(define (450apply fn args)  
  (match fn  
    ...  
    [(? procedure?) ... ] ;; racket function  
    [(lm-result params body env) ;; user-defined function  
     ... (ast-fn body ... ) ... (foldl env-add env params args) ... ]))
```

`run/e : AST Env -> Result`


these are lists

# CS450 Lang “Apply”

```
;; A Result is one of:  
;; - ...  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST Env)  
(struct lm-result [params body env])
```

```
;; 450apply : Result List<Result> -> Result  
(define (450apply fn args)  
  (match fn  
    ...  
    [(? procedure?) ...] ;; racket function  
    [(lm-result params body env) ;; user-defined function  
     (run/e body (foldl env-add env params args))]))
```

run/e : AST Env -> Result



# CS450 Lang “Apply”

```
;; A Result is one of:  
;; - ...  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST Env)  
(struct lm-result [params body env])
```

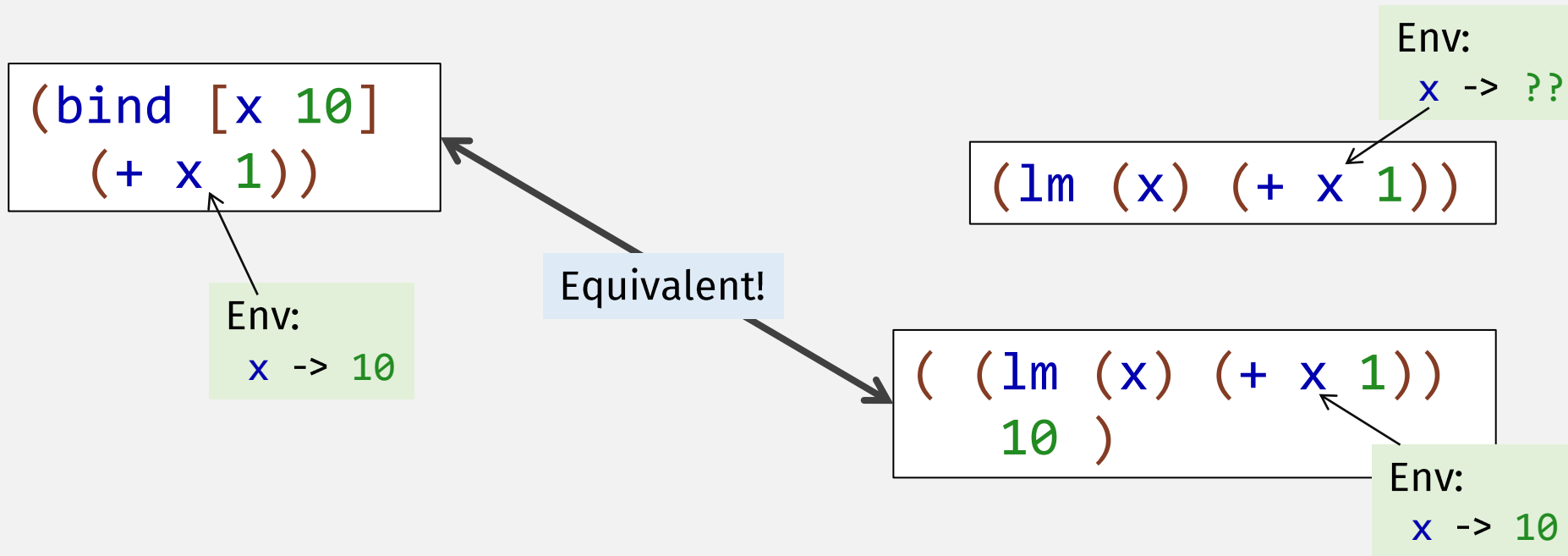
```
;; 450apply : Result List<Result> -> Result  
(define (450apply fn args)  
  (match fn
```

...

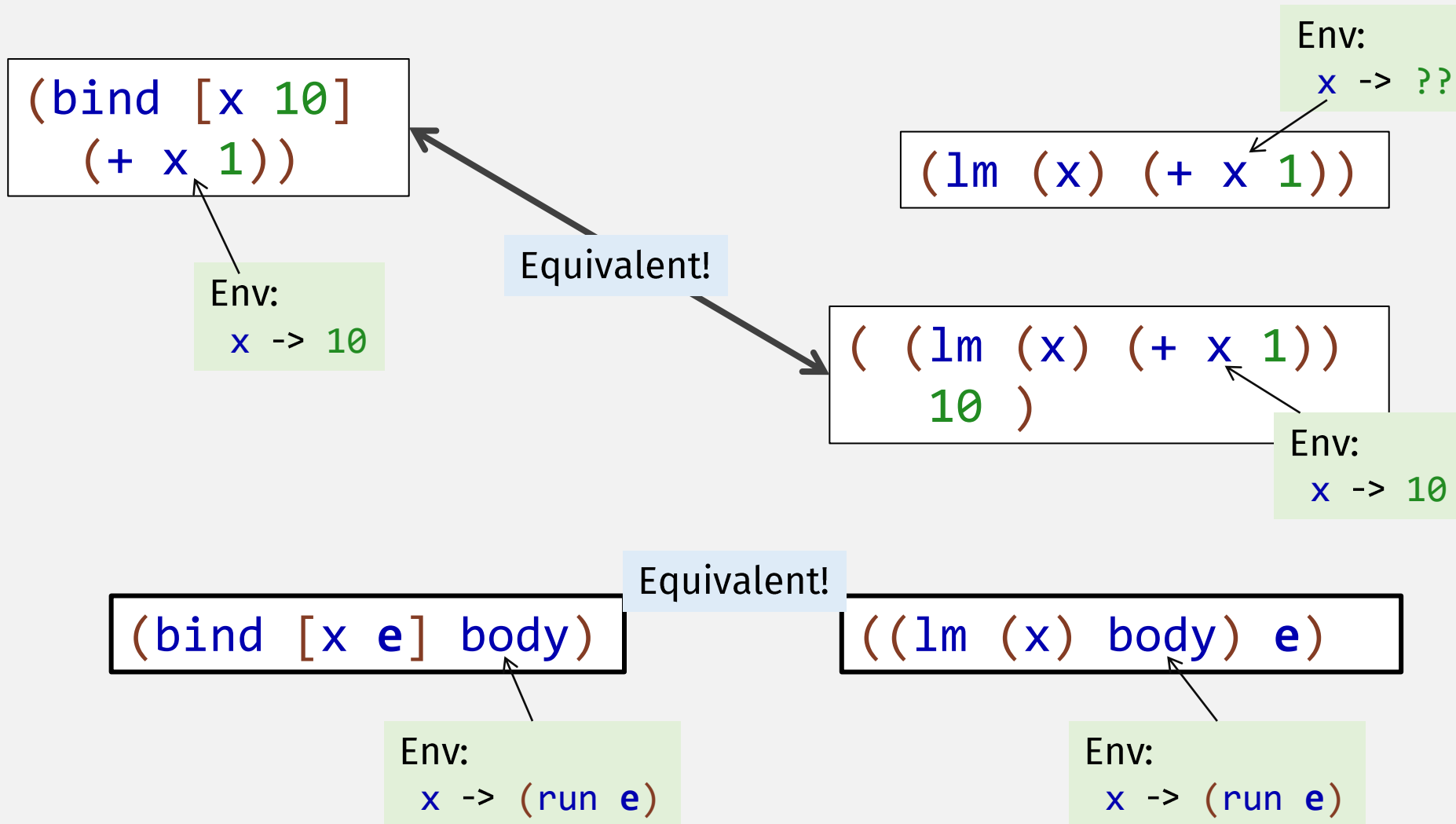
Runs a Racket function

```
[(? procedure?) (apply fn args)] ;; racket function  
[(lm-result params body env)      ;; user-defined function  
 (run/e body (foldl env-add env params args))])])
```

# bind, λm, and their environments



# bind = $\lambda m$ + fn call!



# CS450 Lang “Apply”

```
;; A Result is one of:  
;; - ...  
;; - (Racket) Function  
;; - (mk-lm-res List<Symbol> AST Env)  
(struct lm-result [params body env])
```

```
;; 450apply : Result List<Result> -> Result  
(define (450apply fn args)  
  (match fn  
    ...  
    [(? procedure?) (apply fn args)] ;; racket function  
    [(lm-result params body env)      ;; user-defined function  
     (run/e body (foldl env-add env params args))]))
```

Runs a Racket function

WAIT! What if the the number of params and args don't match!

# CS450 Lang “Apply”

```
;; 450apply : Result List<Result> -> Result
(define (450apply fn args)
  (match fn
    ...
    [(? procedure?) (apply fn args)] ;; racket function
    [(lm-result params body env)      ;; user-defined function
     (if (= (length params) (length args))
          (run/e body (foldl env-add env params args))
          ...
     ]))
```

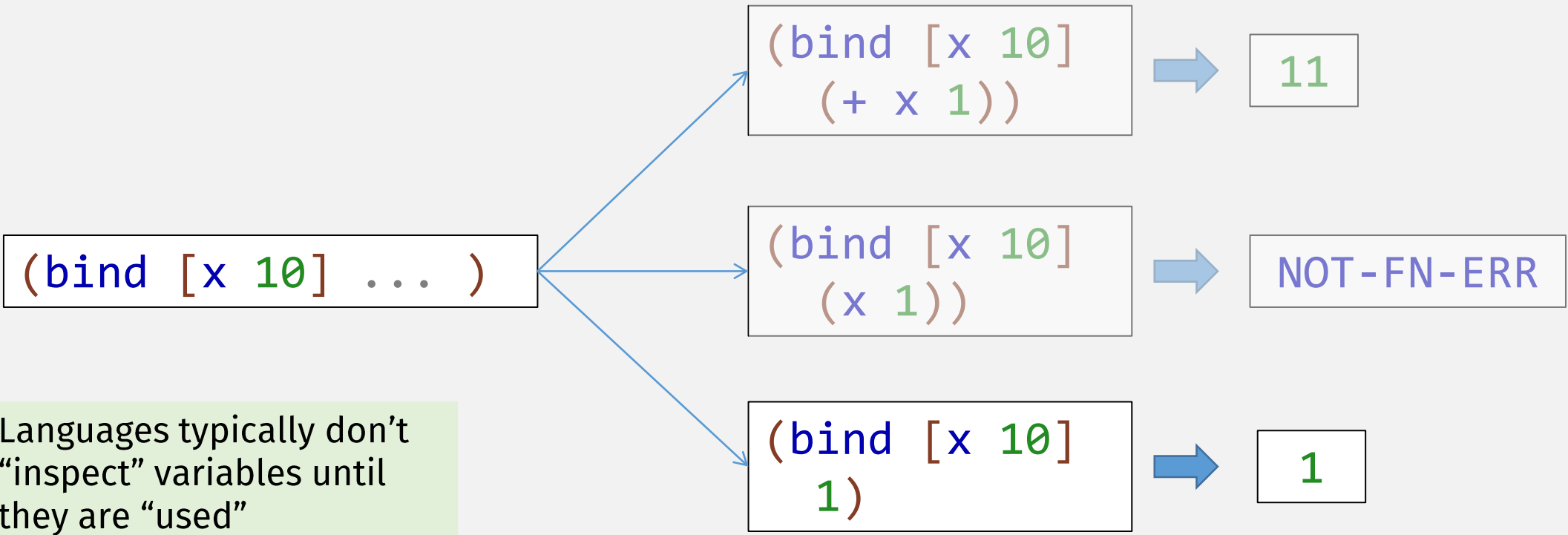
# CS450 Lang “Apply”: arity error

```
;; 450apply : Result List<Result> -> Result
(define (450apply fn args)
  (match fn
    ...
    [(? procedure?) (apply fn args)] ;; racket function
    [(lm-result params body env)      ;; user-defined function
     (if (= (length params) (length args))
         (run/e body (foldl env-add env params args))
         ARITY-ERROR)]))
```

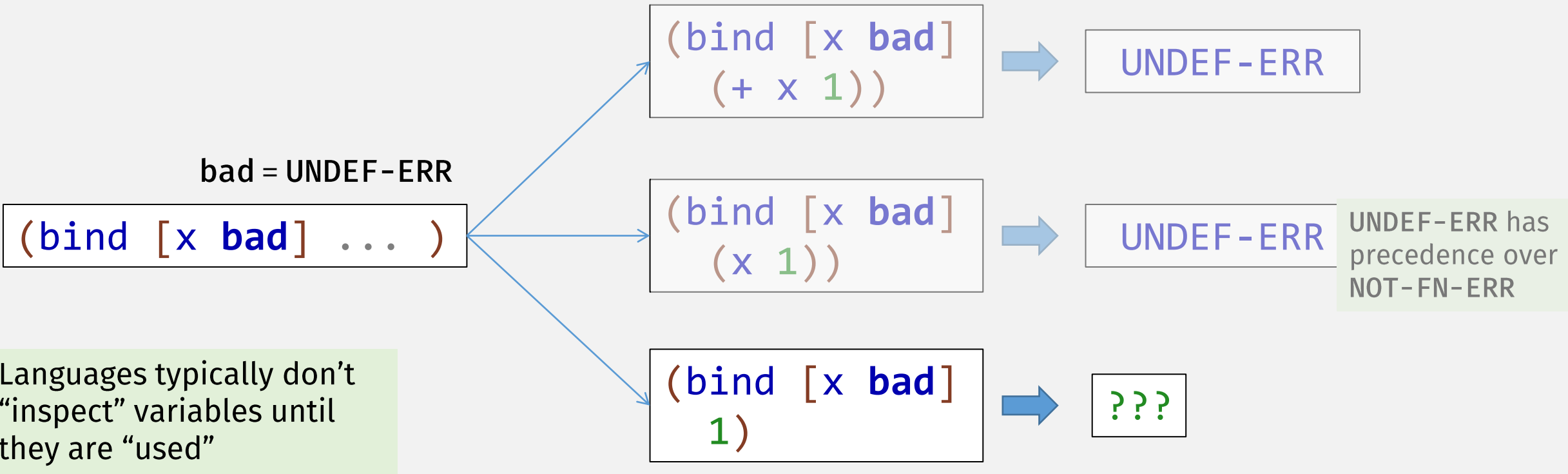
;; An ErrorResult is one of:  
;; - UNDEFINED-ERROR  
;; - NOT-FN-ERROR  
;; - **ARITY-ERROR**

;; A Result is one of:  
;; - Number  
;; - **ErrorResult**  
;; - FnResult

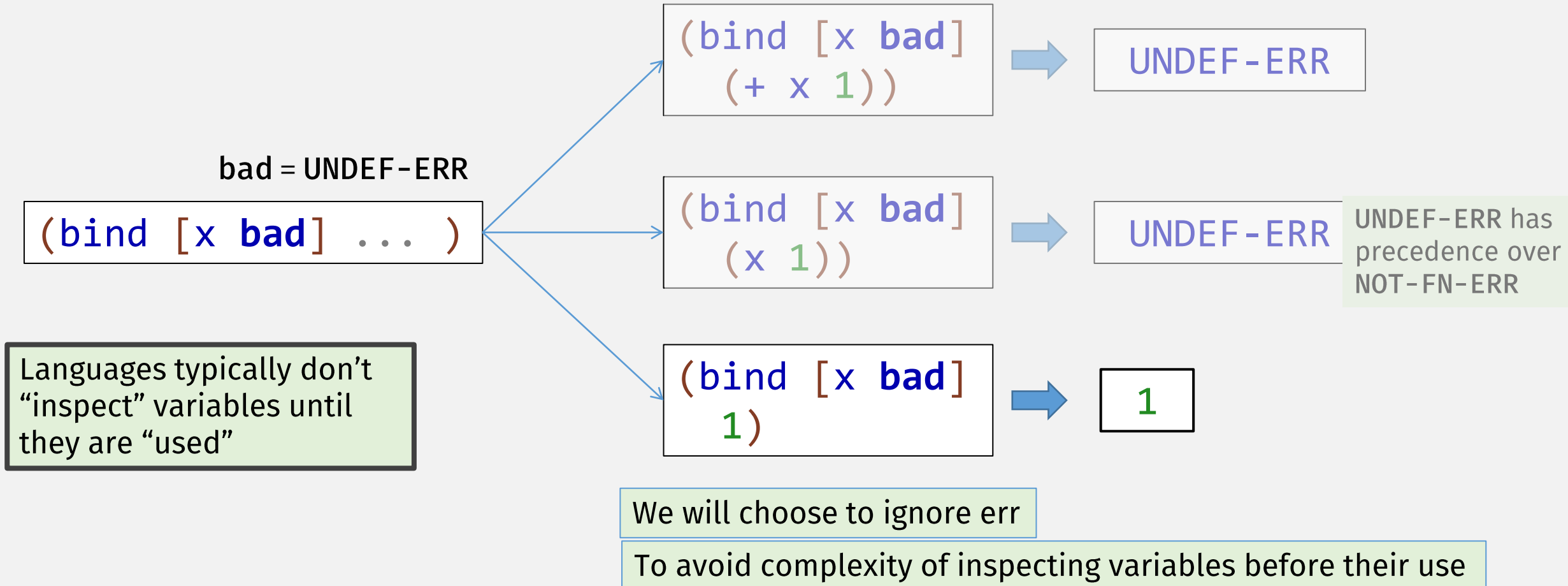
# *Interlude:* Error Propagation Example



# Interlude: Error Propagation Example



# Interlude: Error Propagation Example



# In-class Coding 4/16 `lm` examples

```
(check-equal?
 (eval450
  '(bind [x 10]
        (lm (y) (+ x y)) 20 )))
30 ) ; with bind
```

```
(check-equal?
 (eval450
  '(bind [x 10]
        (lm (y) (+ x y)))
  20 ))
30 ) ; with bind (lm only)
```

Expression that evaluates to a function result

argument → 20

Come up with some of your own!  
(i.e., not my examples)  
(can be error cases, both “syntax” and “result”)

```
(check-equal?
 (eval450
  '(lm (x y) (+ x y))
  10 20 ) )
30 )
```

```
;; A Program is one of:
;; - Atom
;; - Variable (Var)
;; - `(bind [ ,Var ,Program] ,Program)
;; - `(lm ,List<Var> ,Program)
;; - (cons Program List<Program>)
```