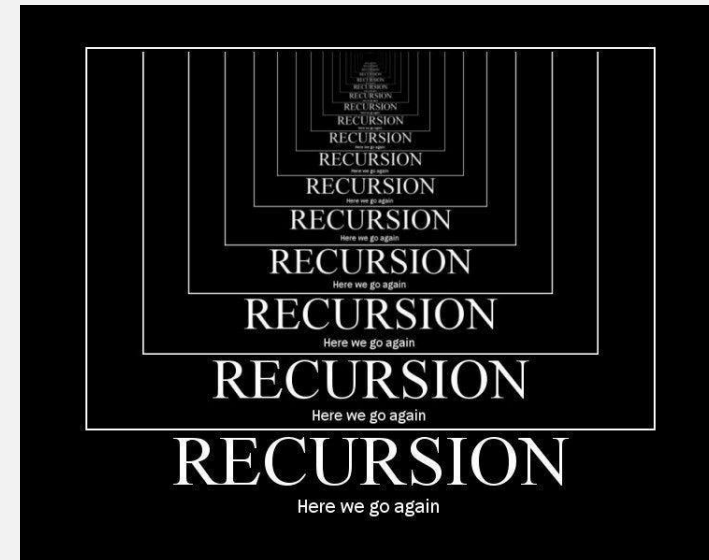


UMass Boston Computer Science
CS450 High Level Languages

Implementing Recursive Variables

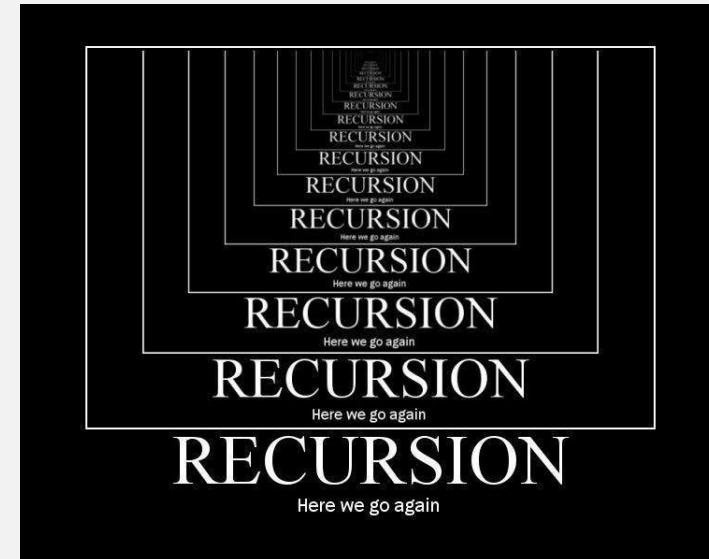
(with mutation??)

Thursday, April 23, 2026



Logistics

- HW 10 in
 - ~~due: Thu 4/23 11am EST~~
- HW 11 out
 - due: Tues 5/5 11am EST



Previously

bind in “CS450” Lang

```
;; A Variable (Var) is a Symbol
```

```
;; A Prog is one of:
```

```
;; ...
```

```
;; - Var
```

```
;; - `(bind [ ,Var ,Prog] ,Prog)
```

```
;; ...
```

Reference a variable binding

new binding is in-scope
(can be referenced) here

Create a new
variable binding

new binding is **not**
in-scope here



Previously

bind examples

```
;; A Prog is one of:  
;; ...  
;; - Var  
;; - `(bind [,Var ,Prog] ,Prog)  
;; ...
```

new binding is **not**
in-scope here

```
(check-equal?  
  (eval450  
    '(bind [x (+ x 20)]  
           x))  
  UNDEFINED-ERROR )
```

???

bind examples, with functions

```
;; A Prog is one of:  
;; ...  
;; - Var  
;; - `(bind [,Var ,Prog] ,Prog)  
;; - `(lm ,List<Var> ,Prog)  
;; - (cons Prog List<Prog>)
```

"lambda"
function

function call

function

arguments

```
(check-equal?  
  (eval450  
    '(bind [f (lm (x) (+ x 4))]  
           (f 6)))  
  10 )
```

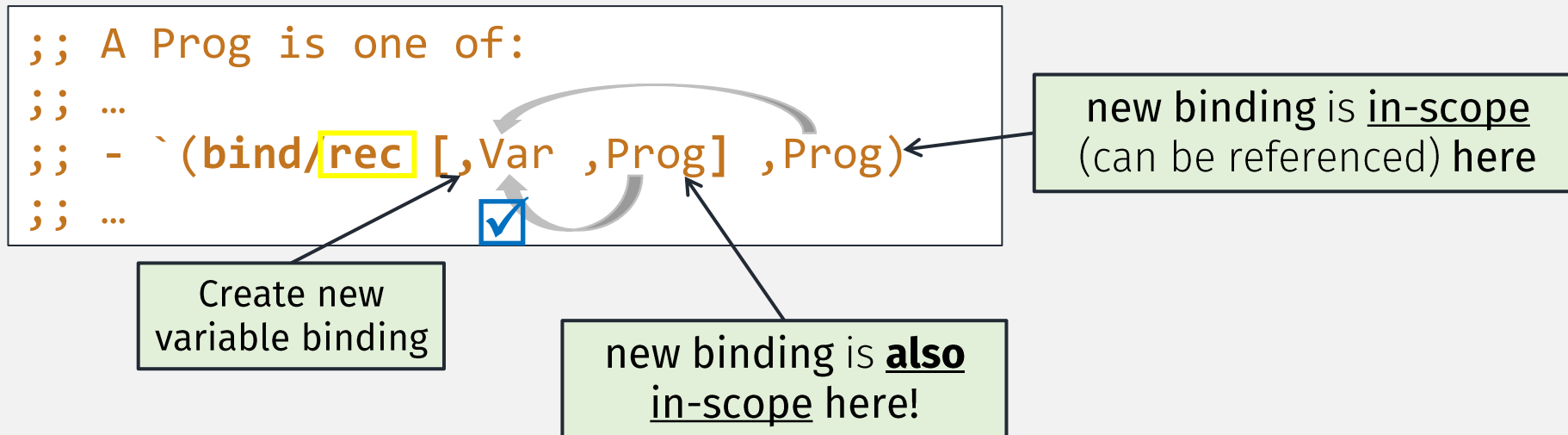
f not in-scope here
(so function can't be recursive!)

```
(check-equal?  
  (eval450  
    '(bind [f (lm (x) (f x))]  
           (f 6)))  
  UNDEF-ERR)
```

f not in-scope here
(so function can't be recursive!)

How to add recursion to our language?

bind/rec in “CS450” Lang

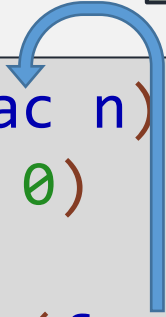


Racket recursive function examples

```
(define (fac n)
  (if (= n 0)
      1
      (* n (fac (- n 1)))))
```

(fac 5) ; => 120

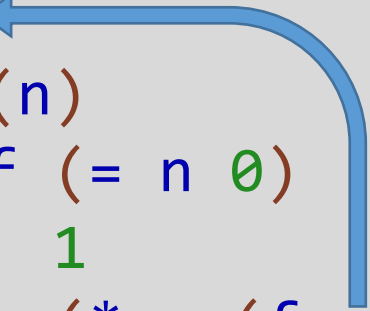
RACKET



Equivalent to ...

```
(letrec
  ([fac
   (lambda (n)
     (if (= n 0)
         1
         (* n (fac (- n 1)))))])
  (fac 5)) ; => 120
```

RACKET



bind/rec examples

```
;; A Prog is one of:  
;; ...  
;; - `(bind/rec [ ,Var ,Prog] ,Prog)  
;; - `(iffy ,Prog ,Prog ,Prog)  
;; ...
```

JS "truthy if" (hw10)

```
(letrec  
  ([fac  
    (λ (n)  
      (if (= n 0)  
          1  
          (* n (fac (- n 1))))))]  
  (fac 5)) ; => 120
```

RACKET

Equivalent to ...

```
(bind/rec  
 [fac  
  (lm (n)  
    (iffy n  
          (* n (fac (- n 1))))  
          1))]  
 (fac 5)) ; => 120
```

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Zero is "falsy" (hw10)

Assume "-"
primitive in
INIT-ENV

RACKET define is lambda

```
(define (f n)  
  (- n 1))
```

RACKET

Equivalent to ...

```
(define f  
  (λ (n)  
    (- n 1)))
```

RACKET

RACKET define is lambda and letrec

```
(define (factorial n)
  (if (= n 0)
      1
      (* n (factorial (- n 1)))))
```

RACKET

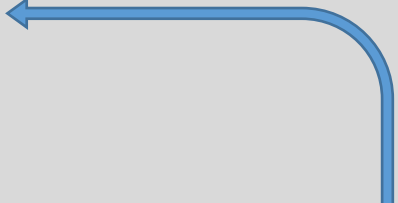
Equivalent to ...

```
(define factorial
  (letrec
    ([fac ← (λ (n)
              (if (= n 0)
                  1
                  (* n (fac (- n 1)))))])
    fac))
```

RACKET

In-class programming: recursion with letrec

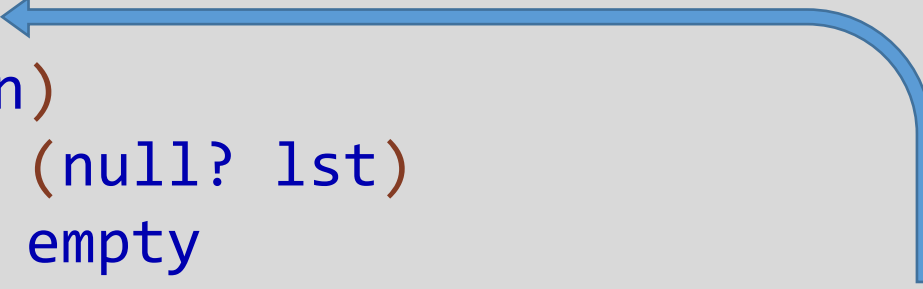
```
(define (map f lst)
  (if (null? lst)
      empty
      (cons (f (first lst)) (map f (rest lst)))))
```



RACKET

Equivalent to ...

```
(define map
  (letrec
    ([_map
     (λ (n)
      (if (null? lst)
          empty
          (cons (f (first lst)) (_map f (rest lst))))))]
    _map))
```



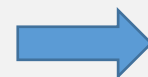
RACKET

Task: re-write your favorite recursive function using (non-function) **define** and **letrec**

Running `bind/rec` programs

```
;; A Prog is one of:  
;; ...  
;; - `(bind/rec [,Var ,Prog] ,Prog)  
;; ...
```

parse



```
;; An AST is one of:  
;; ...  
;; - (mk-recb Symbol AST AST)  
;; ...  
(struct recb [var expr body])
```

run



```
;; A Result is a:  
;; - ...
```

Running `bind/rec` programs

TEMPLATE ?

```
;; run: AST -> Result  
;; Computes result of  
running CS450 Lang AST
```

```
;; An AST is one of:  
;; ...  
;; - (mk-recb Symbol AST AST)  
;; ...  
(struct recb [var expr body])
```

run



```
;; A Result is a:  
;; - ...
```

Running `bind/rec`

TEMPLATE : extract pieces

```
;; run: AST -> Result
```

```
(define (run p)
```

```
  (define (run/e p env)
```

```
    (match p
```

```
      ...
```

```
      [(recb x e body) ?? x ?? e ?? body ]))
```

```
      ... ))
```

```
    (run/e p ??? ))
```

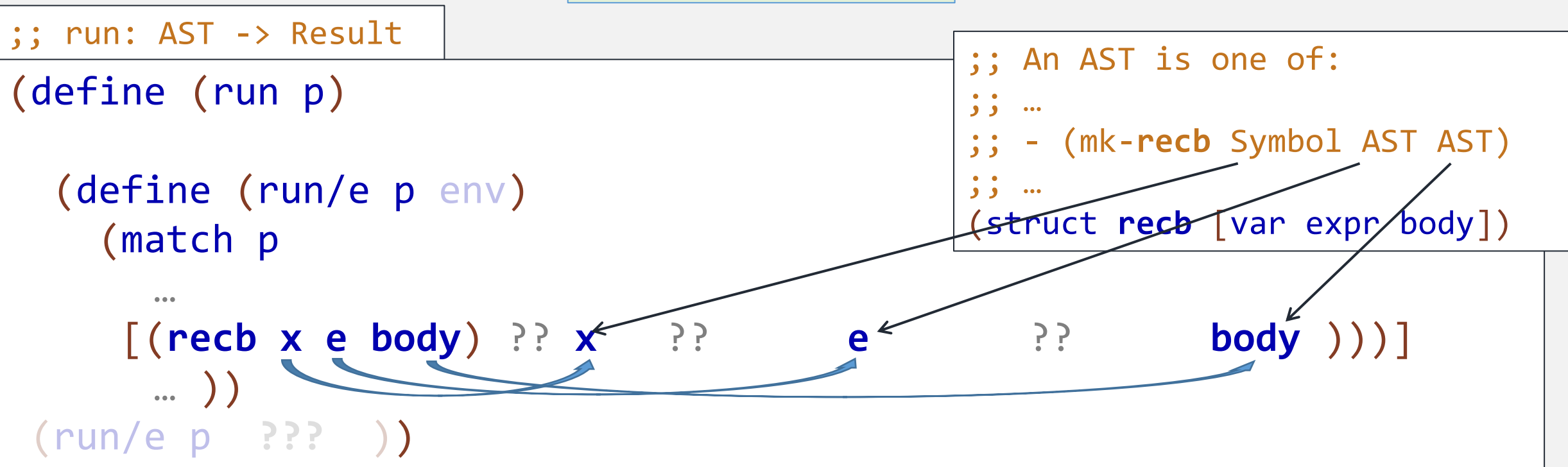
```
;; An AST is one of:
```

```
;; ...
```

```
;; - (mk-recb Symbol AST AST)
```

```
;; ...
```

```
(struct recb [var expr body])
```



Running `bind/rec`

TEMPLATE : recursive call

```
;; run: AST -> Result
```

```
(define (run p)
```

```
  (define (run/e p env)
```

```
    (match p
```

```
      ...
```

```
      [(recb x e body) ?? x ?? (run/e e ??) ?? (run/e body ??) ]
```

```
      ... ))
```

```
    (run/e p ??? ))
```

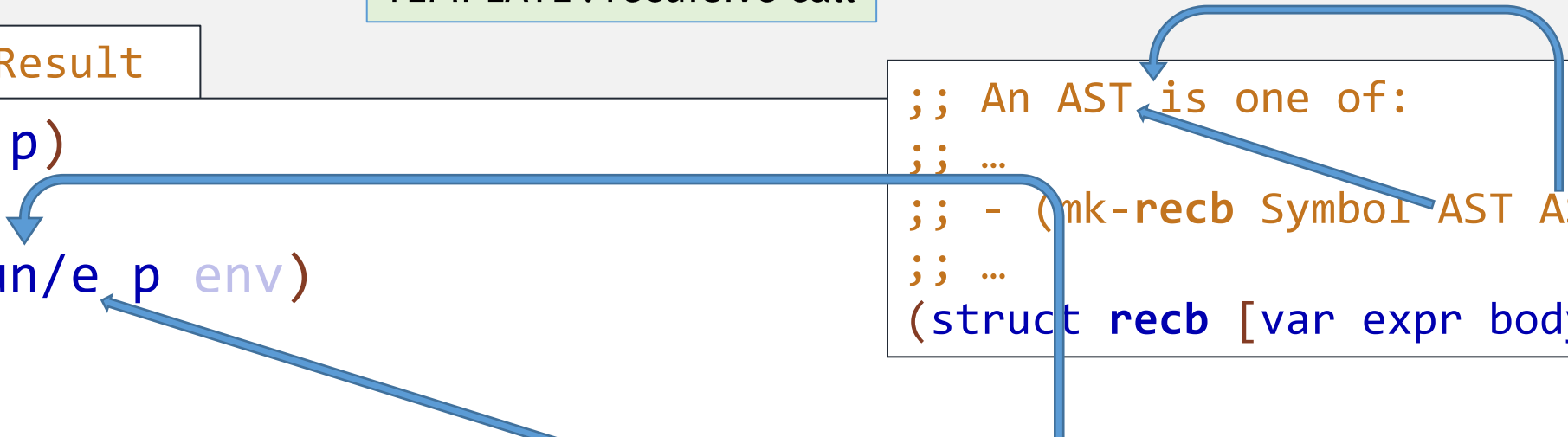
```
;; An AST is one of:
```

```
;; ...
```

```
;; - (mk-recb Symbol AST AST)
```

```
;; ...
```

```
(struct recb [var expr body])
```



Running `bind/rec`, using environment

```
;; run: AST -> Result
```

```
;; An Environment (Env) is one of:  
;; - empty  
;; - (cons (list Var Result) Env)
```

```
(define (run p)
```

```
  ;; accumulator env : Environment
```

```
  (define (run/e p env)
```

```
    (match p
```

```
      ...
```

```
      [(recb x e body) ?? x ?? (run/e e ??) ?? (run/e body ??) ]
```

```
      ... ))
```

```
  (run/e p INIT-ENV ))
```

Running `bind/rec`, using environment

```
;; run: AST -> Result
```

```
(define (run p)
  ;; accumulator env : Environment
  (define (run/e p env)
    (match p
      ...
      [(recb x e body)
       (define env/x (env-add env x (run/e e env)))
       (run/e body env/x)]
      ... ))
  (run/e p INIT-ENV ))
```

2. add x binding to environment

1. Compute Result for x

Running `bind/rec`, using environment

CS450LANG
Recursive Example

`;; run: AST -> Result`

```
(define (run p)
  ;; accumulator env : Environment
  (define (run/e p env)
    (match p
      ...
      [(recb x e body)
       (define env/x (env-add env x (run/e e env/x)
                              (run/e body env/x)))
       ...
      ])
    (run/e p INIT-ENV
```

```
(bind/rec
 [fac ←
  (lm (n)
    (iffy n
      (* n (fac (- n 1)))
      1)))
 (fac 5)) ; => 120
```

??? This is circular! (no base case)

PROBLEM:
x should be in-scope here too!

Compute body
with x in-scope

Interlude: Mutation

- **Mutating** a variable means: to change its value after it is defined

```
(define x 3)
(display x) ; 3
(set! x 5) ; mutate x
(display x) ; 5
```

Interlude: Mutation

- **Mutating** a variable means: to change its value after it is defined
- **Mutation** should be rarely used, only in appropriate situations

Interlude: Mutation

- Mutating a variable means: to change its value after it is defined
- **Mutation** should be rarely used, only in appropriate situations

Item 3: Use const whenever possible.
Effective C++, Scott Meyers, 2005.

Item 15, "Minimize mutability." **Joshua Bloch** Author, *Effective Java, Second Edition*

Joshua Bloch, Google's chief Java architect, is a former Distinguished Engineer at Sun Microsystems, where he led the design and implementation of numerous Java platform features, including JDK 5.0 language enhancements and the award-winning Java Collections Framework.

Immutability
makes code
easier to read
and understand

Item 15 tells you to keep the state space of each object as simple as possible. If an object is immutable, it can be in only one state, and you win big. You never have to worry about what state the object is in, and you can share it freely, with no need for synchronization. If you can't make an object immutable, at least minimize the amount of mutation that is possible. This makes it easier to use the object correctly.

Interlude: Mutation

- **Mutating** a variable means: to change its value after it is defined
- **Mutation** should be rarely used, only in appropriate situations

Because:

- It makes code more difficult to read and understand
 - (just like other “bad” features, e.g., inheritance and dynamic scope)

- It violates “Separation of concerns”

```
(define x 3)
(do-something x) ; mutate x??
(display x) ; ???
```

Interlude: Mutation

- **Mutating** a variable means: to change its value after it is defined
- **Mutation** should be rarely used

When is using **mutation** ok:

- **Performance**
 - Typically **not using high-level languages!** (OS, AAA game i.e., not this class!)
 - Beware of **pre-mature optimization** (don't optimize without hard profiling data)!
- **Shared state** (in distributed programs)
 - Beware of **race conditions and deadlock!**
- **Circular data structures** (e.g., circular lists)

Running `bind/rec`, recursive environment items

```
;; run: AST -> Result
```

```
(define (run p)
  (define (run/e p env)
    (match p
      ...
      [(recb x e body)
       (define env/x (env-add env x (run/e e env/x)))
       (run/env body env/x)]
      ...
    ))
  (run/e p INIT-ENV))
```

??? This is **circular!** (no base case)

env/x

PROBLEM:
x should be in-scope here too!

Compute body
with x in-scope

Running `bind/rec`, recursive environment items

```
;; run: AST -> Result
```

```
(define (run p)
  (define (run/e p env)
    (match p
      ...
      [(recb x e body)
       (define placeholder (box CIRCULAR-ERROR))
       (define env/x (env-add env x placeholder))

       (run/env body env/x)]
      ... ))
  (run/e p INIT-ENV ))
```

Creates mutable box.
Makes mutation explicit

```
;; A Result is a:
;; - Number
;; - FunctionResult
;; - ErrorResult
```

```
;; An ErrorResult is a:
;; - UNDEFINED-ERROR
;; - ARITY-ERROR
;; - CIRCULAR-ERROR
```



Running `bind/rec`, recursive environment items

```
;; run: AST -> Result
```

```
(define (run p)
```

```
  (define (run/e p env)
```

```
    (match p
```

```
      [(recb x e body)
```

```
        (define placeholder (box CIRCULAR-ERROR))
```

```
        (define env/x (env-add env x placeholder))
```

```
        (run/env body env/x)]
```

```
      ... ))
```

```
(run/e p INIT-ENV ))
```

```
;; An Environment (OLD) (Env) is one of:
```

```
;; - empty
```

```
???
```

```
;; - (cons (list Var Result) Env)
```

(how would `env-add`
and `env-lookup`
need to change?)

```
;; An Environment (Env) is a: Listof<(list Var EnvVal)>
```

```
;; An EnvVal is one of:
```

```
;; - Result
```

```
;; - Box<Result>
```

env/x

...	...
x	CIRCULAR-ERROR

Running `bind/rec`, recursive environment items

CS450LANG

```
(bind/rec [f f] f)  
; => CIRCULAR-ERROR
```

```
;; run: AST -> Result
```

```
(define (run p)  
  (define (run/e p env)  
    (match p ...  
      [(recb x e body)  
       (define placeholder (box CIRCULAR-ERROR))  
       (define env/x (env-add env x placeholder))  
       (define x-result (run/env e env/x))  
       (run/env body env/x)]  
      ... ))  
  (run/e p INIT-ENV))
```

Compute `x`'s
Result with
`x` in-scope!

Non-function, circular recursive
references (no base case)
produce error results!

env/x

...	...
x	CIRCULAR-ERROR

Running `bind/rec`, recursive environment items

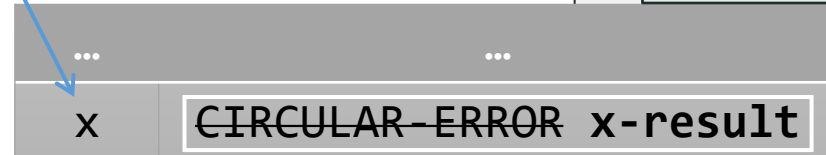
```
;; run: AST -> Result
```

```
(define (run p)
  (define (run/e p env)
    (match p
      ...
      [(recb x e body)
       (define placeholder (box CIRCULAR-ERROR))
       (define env/x (env-add env x placeholder))
       (define x-result (run/env e env/x))
       (set-box! placeholder x-result)
       (run/env body env/x)]
      ... ))
  (run/e p INIT-ENV ))
```

Close the (circular data structure) loop, with **mutation!**

Explicitly mutate mutable box

env/x



Running `bind/rec`, recursive environment items

CS450LANG

```
;; run: AST -> Result
```

```
(define (run p)
  (define (run/e p env)
    (match p
      ...
      [(recb x e body)
       (define placeholder (box CIRCULAR-ERROR))
       (define env/x (env-add env x placeholder))
       (define x-result (run/env e env/x))
       (set-box! placeholder x-result)
       (run/env body env/x)]
      ...
    ))
  (run/e p INIT-ENV ))
```

Compute body with x in-scope

```
(bind/rec
 [fac
  (lm (n)
    (iffy n
      (* n (fac (- n 1))
        1)))])
(fac 5) ; => 120
```



env/x

