Goals
This assignment aims to help you:

- Learn about dynamic programming.
- Compression and coding

Reading
K&T, Chapter 4 (greedy algorithms), Chapter 6 (dynamic programming) S&W, Chapter 5.5 (data compression)

Questions

1. Change making problem: As a reminder, the problem is defined as follows: Given coin denominations and an amount to be paid, devise a method to pay that amount using the fewest possible number of coins. We showed that the greedy algorithm always produces an optimal solution for US coins, that is – \{1, 5, 10, 25, 100\} cents. However, for other denominations, this greedy cashier’s strategy might not be optimal. For example, if we add a 7 cent coin, the optimal solution for 14 cents is two coins of 7, but the greedy algorithm gives the following: \{10, 1, 1, 1, 1\}.

In this problem you are asked to implement the dynamic programming solution to the coin changing problem that always produces an optimal solution (i.e., minimum number of coins) regardless of the coin denominations. This problem was addressed by Prof. Ouyang and myself in class.

Your input is:

- An array denominations[1..n] containing the n coin denominations \(d_1, ..., d_n\) that you can use (for example, \{1, 10, 21, 34, 70, 100\}, in this case \(n=6\)).
- The amount \(M\) that you need to pay (in the example above, \(M = 140\) cents).

You can assume that all the coin denominations and \(M\) are positive integer numbers.

The input format is a file coins.txt where the first line is the amount of money you should count and the second line is the denominations, separated by spaces. For the example above, coins.txt looks like this:

```
140
1 10 21 34 70 100
```

You can assume that this array is already sorted in increasing order (with no repetitions, of course). You can also assume that you have an unlimited number of coins of each denomination.

Usage: cs310.Coins < coins.txt

Your output should be:
• The minimum number of coins needed to make change for M (in the example above where M=140, the optimal number of coins is 2).

• The number of coins of each denomination used in the optimal solution (in the example above where M=140, the optimal answer is 2 coins of 70 cents. Remember that in the general case more than one denomination can be used.

**Hint:** You should keep track of the solution for any \( n \leq M \), so you should define an array of size \( M \) that keeps track of the number of coins for each \( n \), plus another array of size \( M \) that keeps track of the last coin used, so you can trace back the optimal combination of coins.

Please use standard Java, not S&W’s library for this question.

2. **Sequence alignment:** The question is based on “Creative Programming assignments” by Sedgewick and Wayne:

[https://introcs.cs.princeton.edu/java/assignments/sequence.html](https://introcs.cs.princeton.edu/java/assignments/sequence.html)

Adaptations are taken from: [http://www.cis.upenn.edu/ cis110/12su/hw/hw06/dynprog.shtml](http://www.cis.upenn.edu/ cis110/12su/hw/hw06/dynprog.shtml)

The background in the question provides the recursive brute force process, but you should only implement the dynamic programming part. You may test your program on the sequences suggested in the checklist (see link to original assignment).

It is very important that you fully understand the dynamic programming sequence alignment algorithm before you start programming. The algorithm, once you understand it, is very straightforward to implement, but the understanding part requires careful consideration. What helps is to run an example on paper. For example, for the two sequences in the question build the dynamic programming matrix and fill it manually, making sure you’re getting what you suppose to get. Notice that the matrix on the webpage is filled out from bottom to top and from right to left (which I find odd, I usually do it backwards as in my class notes). Therefore, the final edit distance can be obtained from opt[0][0]. Please follow the convention in the assignment, and not in the class notes. The idea is the same, you just need to fill the matrix differently.

Some guidelines:

• Your program should get a file from the standard input. Usage: java cs310.EditDistance < input.txt, where input.txt contains two DNA sequences separated by a newline. This is different from the description in both links above.

• Your program should consist of a class Match.java with one static method to compute the optimal match between a pair of objects. The optimal solution is returned as a linked list of Path nodes. Each Path node stores the row and column of the element it represents, the total cost of the match from that point to the end, and a pointer to the next path node. So for opt matrix above, the Path nodes would follow the red numbers: the row and col variables of the first node would both be 0 and cost would be 7. In the second node, row and col would be 1 and cost would be 6, and so on. The reason for this is that the DP formula gives you the edit distance, but you also need to recover the alignment itself.

• The classes Match, Path and EditDistance should follow the API described here: [http://www.cis.upenn.edu/ cis110/12su/hw/hw06/dynprog.shtml](http://www.cis.upenn.edu/ cis110/12su/hw/hw06/dynprog.shtml)

• The output should be in the same format as described in the link above.

• Use standard java and place all your files (from this and the other questions in a cs310 package, as per the instructions in the previous PAs.

In your memo.txt answer the following questions:

(a) What is the output from matching the two sequences:
   \( x = "AACAGTTACC" \) and \( y = "TAAGGTTCA" \)?

(b) What is the runtime of the DP algorithm?

(c) What is the space required to implement the algorithm?
Binary input and binary output: To enable your programs to work with binary data, you will use BinaryStdIn and BinaryStdOut, which are described in Algorithms, 4th edition and part of algs4.jar. To display the binary output when debugging, you can use HexDump, which takes a command-line argument n, reads bytes from standard input and writes them to standard output in hexadecimal, n per line.

> more abra.txt
ABRACADABRA!

41 42 52 41 43 41 44 41 42 52 41 21
96 bits

Move-to-front encoding and decoding: The main idea of move-to-front encoding is to maintain an ordered sequence of the characters in the alphabet, and repeatedly read in a character from the input message, print out the position in which that character appears, and move that character to the front of the sequence. As a simple example, if the initial ordering over a 6-character alphabet is A B C D E F, and we want to encode the input CAAABCCCACCF, then we would update the move-to-front sequences as follows:

<table>
<thead>
<tr>
<th>move-to-front</th>
<th>in</th>
<th>out</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B C D E F</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>C A B D E F</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>A C B D E F</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>A C B D E F</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>A C B D E F</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>B A C D E F</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>C B A D E F</td>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>C B A D E F</td>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>C B A D E F</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>A C B D E F</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>C A B D E F</td>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>C A B D E F</td>
<td>F</td>
<td>5</td>
</tr>
<tr>
<td>F C A B D E</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If the same character occurs next to each other many times in the input, then many of the output values will be small integers, such as 0, 1, and 2. The extremely high frequency of certain characters makes an ideal scenario for Huffman coding (which you don’t have to implement).

Move-to-front encoding: Your task is to maintain an ordered sequence of the 256 extended ASCII characters. Initialize the sequence by making the $i^{th}$ character in the sequence equal to the $i^{th}$ extended ASCII character. Now, read in each 8-bit character $c$ from standard input one at a time, output the 8-bit index in the sequence where $c$ appears, and move $c$ to the front.

41 42 52 02 44 01 45 01 04 04 02 26
96 bits

Move-to-front decoding: Initialize an ordered sequence of 256 characters, where extended ASCII character $i$ appears $i^{th}$ in the sequence. Now, read in each 8-bit character $i$ (but treat it as an integer between 0 and 255) from the standard input one at a time, write the $i^{th}$ character in the sequence, and move that character to the front. Check that the decoder recovers any encoded message.

In other words, Move-to-front should be completely reversible.

> java -cp ../lib/algs4.jar cs310.MoveToFront - < abra.txt | java -cp ../lib/algs4.jar cs310.MoveToFront +
ABRACADABRA!

Name your program MoveToFront.java and organize it using the following API:
public class MoveToFront {
    // apply move-to-front encoding,
    // reading from standard input and writing to standard output
    public static void encode()

    // apply move-to-front decoding,
    // reading from standard input and writing to standard output
    public static void decode()

    // if args[0] is '-', apply move-to-front encoding
    // if args[0] is '+', apply move-to-front decoding
    public static void main(String[] args)
}

Performance requirements: The running time of move-to-front encoding and decoding should be proportional to \( R \times n \) (or better) in the worst case and proportional to \( n + R \) (or better) in practice on inputs that arise when compressing typical English text, where \( n \) is the number of characters in the input and \( R \) is the alphabet size.

Delivery

- Match.java, Path.java, EditDistance.java: Usage “java cs310.EditDistance < input.txt”, where input.txt contains two sequences, each in a separate line.
- Coin.java. Usage: java cs310.Coin < coins.txt as described above.
- MoveToFront.java, usage: See above.
- memo.txt . Indicate any late days and answer the questions above.