1. Solve the entire second midterm. Fresh copy attached.

2. **Max-Flow-Min-Cut:** Given the following flow network on which an s-t flow has been computed. The capacity of each edge appears as a label on the edge, and the numbers in parentheses give the amount of flow sent on each edge. (Edges without parentheses – specifically, the four edges of capacity 3 – have no flow being sent on them.)

   ![Flow Network Diagram]

   (a) What is the value of this flow? Is this a maximum (s,t) flow in this graph?
   (b) Find a minimum s-t cut in the flow network and also say what its capacity is.

3. Decide whether the following statement is true or false. If it is true, give a short explanation. If it is false, give a counter example:

   Given an arbitrary flow network, with a source s, a sink t, and a positive integer capacity $c_e$ on every edge $e$; and let $(A, B)$ be a minimum $s-t$ cut with respect to these capacities $\{c_e | e \in E\}$. Now suppose we add 1 to every capacity; then $(A, B)$ is still a minimum $s-t$ cut with respect to these new capacities $\{1 + c_e | e \in E\}$.

4. Exercise 1.1 (page 3 in the NP notes).

5. Exercise 2.1 (page 9)

6. Exercise 3.6 (page 16)

7. Problem name: HITTING SET

   Instance: A collection $C$ of subsets of a set $S$ together with a positive integer $K$.

   Question: Does $S$ contain a hitting set for $C$ of size $K$ or less – that is, a subset $S' \subseteq S$ with $|S'| \leq K$ and such that $S'$ contains at least one element of each set $c \in C$?

   Prove that HITTING SET is NP-complete.

   To do this you need to do two things:

   (a) Prove that HITTING SET is in NP. This should be extremely easy.
(b) Prove that some problem that is already known to be NP-complete polynomially reduces to HITTING SET.

Hint: In this case, I suggest you prove that $VC \leq_P$ HITTING SET.

8. A dominating set for a graph $G = (V, E)$ is a subset $D$ of $V$ such that every vertex not in $D$ is adjacent to at least one member of $D$. Here is an example (the dominating set is colored in black). Question: Does $G$ contain a dominating set of size $K$ or less?

(a) Show that the dominating set is in NP

(b) Show that it is NP complete by constructing a reduction from Vertex Cover (these are indeed similar problems).