CS724: Topics in Algorithms
Problem Set 2

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Problem 1:

Compute the Stirling numbers of the first kind of the form $s(4, k)$ for $1 \leqslant k \leqslant 4$.

Solution 1:

Striling numbers of the first kind $s(4, k)$ are the coefficients of x in the polynomial

$$(x)_4 = x(x-1)(x-2)(x-3) = x^4 - 6x^3 + 11x^2 - 6x,$$

so they are $1, -6, 11$, and -6.

Problem 2:

How many partitions having three blocks of a set having 5 elements exist?

Solution 2:

The required number is $S(5,3)$, where Ss are the Sterling numbers of the second kind. Therefore, we have:

$$
\begin{aligned}
S(5,3) &= \frac{1}{3!}\left(\sum_{j=0}^{2}(-1)^j\binom{3}{j}(-1)^j(3-j)^5\right) \\
&= \frac{1}{6}\left(\binom{3}{0}(-1)^0(3-0)^5 + \binom{3}{1}(-1)^1(3-1)^5 + \binom{3}{2}(-1)^2(3-2)^5\right) \\
&= \frac{1}{6}\left(3^5 - 3\cdot 2^5 + 3\right) \\
&= \frac{1}{6}\left(243 - 96 + 3\right) = 25.
\end{aligned}
$$

So, there are 25 partitions with three blocks of a set having 5 elements.