THEORY OF COMPUTATION
Problem session - 13

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UMB
1 Problems
Computations in $S_n$ involve strings directly and make use of

\[ V \leftarrow \sigma V \]
\[ V \leftarrow V^- \]
\[ \text{IF } V \text{ ENDS } \sigma \text{ GOTO } L \]
Also, the macros

\[
\begin{align*}
\text{IF } V \neq 0 & \text{ GOTO } L \\
V & \leftarrow 0 \\
& \text{GOTO } L \\
\text{IF } V \text{ ENDS } s_i & \text{ GOTO } B_i \\
V' & \leftarrow V,
\end{align*}
\]

as well as any other macro for which you provide the appropriate expansion in \( S_n \).
Problem 1: Write an expansion in $S_n$ for the macro $V \leftarrow V\sigma$, which means placing the symbol $\sigma$ to the right of the string which is the value of $V$. 
Solution of Problem 1: The difficulty of this problem is that symbols are added to the left of strings in $S_n$ and we wish to add symbols to the right. The following is an expansion of $V \leftarrow V\sigma$. 
\[ [A_i] \quad \text{IF } V \text{ ENDS } s_i \text{ GOTO } B_i (1 \leq i \leq n) \]
\[ V \leftarrow \sigma V \]
\[ \text{GOTO } D \]

\[ [B_i] \quad W \leftarrow s_i W \]
\[ V \leftarrow V^- \]
\[ \text{GOTO } A_i \]

\[ [D_i] \quad \text{IF } W \text{ ENDS } s_i \text{ GOTO } C_i (1 \leq i \leq n) \]
\[ \text{GOTO } E \]

\[ [C_i] \quad W \leftarrow W^- \]
\[ V \leftarrow s_i V \]

\begin{tabular}{|c|c|}
\hline
\( V \) & \( W \) \\
\hline
\( s_1 s_2 s_3 \) & \( s_3 \) \\
\( s_1 s_2 \) & \( s_2 s_3 \) \\
\( s_1 \) & \( s_1 s_2 s_3 \) \\
\( 0 \) & \( s_1 s_2 s_3 \) \\
\( s \) & \( s_1 s_2 \) \\
\( s_3 s \) & \( s_1 s_2 \) \\
\( s_2 s_3 s \) & \( s_1 \) \\
\( s_1 s_2 s_3 s \) & \( 0 \) \\
\hline
\end{tabular}
Problem 2: Show that the function $f(x) = x^R$ is computable in $S_n$. This should allow us to use the macro $V \leftarrow V^R$. 
Solution to Problem 2: In all cases, lines labeled with an indexed label go from 1 to \( n \).

\[
\begin{array}{ll}
[A_i] & \text{IF } V \text{ ENDS } s_i \text{ GOTO } B_i \\
& \text{GOTO } C \\
[B_i] & W \leftarrow s_i W \\
& V \leftarrow V^- \\
& \text{GOTO } A_1 \\
[C] & \text{IF } W \text{ ENDS } s_i \text{ GOTO } D_i \\
& W \leftarrow W^- \\
& \text{GOTO } E \\
[D_i] & V \leftarrow V s_i
\end{array}
\]

\[
\begin{array}{cc}
V & W \\
& s_s s_s s_3 0 \\
& s_1 s_2 s_3 \\
& s_1 s_2 s_3 \\
& 0 s_1 s_2 s_3 \\
& s_3 s_1 s_2 \\
& s_3 s_2 s_1 0 \\
\end{array}
\]
Problem 3: Write an expansion in $S_n$ for the macro $V \leftarrow \neg V$, which means erasing the first symbol of the string which is the value of $V$. 
Solution for Problem 3: Clearly, $V \leftarrow \neg V$ is equivalent to $((x^R)^-)^R$. 