1 Problems

2 Solutions
1. Write a program fragment that exchanges the value of two variables. In other words, if $Z_1 = a$ and $Z_2 = b$, after executing this fragment we have $Z_1 = b$ and $Z_2 = a$.

2. Write a function in $S$ that computes the remainder of the division of $m$ by $n$.

3. Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be the function defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is even}, \\ 0 & \text{if } x \text{ is odd}. \end{cases}$$

Write a program in $S$ that computes $f$.

4. Let $f$ be a partial function such that $f(x) = 1$ if $x$ is even, and $f(x) \uparrow$ if $x$ is odd. Write a program in $S$ that computes $f$. 


Problem 1: Write a program fragment that exchanges the value of two variables. In other words, if \( Z_1 = a \) and \( Z_2 = b \), after executing this fragment we have \( Z_1 = b \) and \( Z_2 = a \).

The following program fragment \( Q(Z_1, Z_2) \) solves the problem:

\[
\begin{align*}
Z_2 & \leftarrow Z_1 + Z_2 \\
Z_1 & \leftarrow Z_2 - Z_1 \\
Z_2 & \leftarrow Z_2 - Z_1
\end{align*}
\]

An example of the sequence of states of this fragment:

<table>
<thead>
<tr>
<th>( Z_1 )</th>
<th>( Z_2 )</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td>initial state</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>after executing ( Z_2 \leftarrow Z_1 + Z_2 )</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>after executing ( Z_1 \leftarrow Z_2 - Z_1 )</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>after executing ( Z_2 \leftarrow Z_2 - Z_1 ).</td>
</tr>
</tbody>
</table>
Problem 2: Write a function in $S$ that computes the remainder of the division of $m$ by $n$.
Examples: $f(20, 7) = 6$, $f(7, 20) = 7(7 = 20 \times 0 + 7)$.

\[
\begin{align*}
[B] & \quad \text{IF } (X_1 \leq X_2) \quad \text{GOTO A} \\
& \quad X_1 \leftarrow X_1 - X_2 \\
& \quad \text{GOTO B} \\
[A] & \quad Y \leftarrow X_1 \\
& \quad \text{GOTO E}
\end{align*}
\]

An example of the sequence of states of this fragment:

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>
Problem 3: Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be the function defined by

$$f(x) = \begin{cases} 
1 & \text{if } x \text{ is even}, \\
0 & \text{if } x \text{ is odd}.
\end{cases}$$

Write a program in $S$ that computes $f$.
Solution:

IF \( X = 0 \) GOTO \( B \)
IF \( X = 1 \) GOTO \( E \)

\[ [A] \quad X \rightarrow X - 2 \]
IF \( X > 2 \) GOTO \( A \)
IF \( X = 0 \) GOTO \( B \)
\( Y \leftarrow 0 \)
GOTO \( E \)

\[ [B] \quad Y \leftarrow 1 \]
Problem 4: Let $f$ be a partial function such that $f(x) = 1$ if $x$ is even, and $f(x) \uparrow$ if $x$ is odd. Write a program in $S$ that computes $f$.

Solution:

IF $X = 0$ GOTO $B$
IF $X = 1$ GOTO $C$

[A] $X \longrightarrow X - 2$
IF $X > 2$ GOTO $A$
IF $X = 0$ GOTO $B$

[C] GOTO $C$

[B] $Y \leftarrow 1$