1 Problems

2 Solutions
1. Write a program fragment that exchanges the value of two variables. In other words, if $Z_1 = a$ and $Z_2 = b$, after executing this fragment we have $Z_1 = b$ and $Z_2 = a$.

2. Write a function in $S$ that computes the remainder of the division of $m$ by $n$.

3. Let $gcd(x_1, x_2)$ be the greatest common divisor of $x_1$ and $x_2$. Write a program in $S$ that computes $f$.

4. Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be the function defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is even}, \\ 0 & \text{if } x \text{ is odd}. \end{cases}$$

Write a program in $S$ that computes $f$.

5. Let $f$ be a partial function such that $f(x) = 1$ if $x$ is even, and $f(x) \uparrow$ if $x$ is odd. Write a program in $S$ that computes $f$. 
Problem 1: Write a program fragment that exchanges the value of two variables. In other words, if $Z_1 = a$ and $Z_2 = b$, after executing this fragment we have $Z_1 = b$ and $Z_2 = a$.

The following program fragment $Q(Z_1, Z_2)$ solves the problem:

\[
\begin{align*}
Z_2 & \leftarrow Z_1 + Z_2 \\
Z_1 & \leftarrow Z_2 - Z_1 \\
Z_2 & \leftarrow Z_2 - Z_1
\end{align*}
\]

An example of the sequence of states of this fragment:

<table>
<thead>
<tr>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td>initial state</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>after executing $Z_2 \leftarrow Z_1 + Z_2$</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>after executing $Z_1 \leftarrow Z_2 - Z_1$</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>after executing $Z_2 \leftarrow Z_2 - Z_1$.</td>
</tr>
</tbody>
</table>
Problem 2: Write a function in $S$ that computes the remainder of the division of $m$ by $n$.

Examples: $f(20,7) = 6$, $f(7,20) = 7(7 = 20 \times 0 + 7)$.

\[
\begin{aligned}
[B] & \quad \text{IF} \ (X_1 \leq X_2) \ \text{GOTO} \ A \\
& \quad X_1 \leftarrow X_1 - X_2 \\
& \quad \text{GOTO} \ B \\
[A] & \quad Y \leftarrow X_1 \\
& \quad \text{GOTO} \ E
\end{aligned}
\]

An example of the sequence of states of this fragment:

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>
Problem 3: The greatest common divisor $gcd(m, n)$ can be computed as follows in Python in a non-recursive manner:

```python
>>> def gcd(m,n):
    while n > 0:
        m = m % n
        p = m
        m = n
        n = p
    return m
```
\[ Z_1 \leftarrow X_1 \]
\[ Z_2 \leftarrow X_2 \]
\[ [B] \quad \text{IF } Z_2 = 0 \text{ GOTO A} \]
\[ Z_1 \leftarrow f(Z_1, Z_2) \]
\[ Q(Z_1, Z_2) \]
\[ \text{GOTO B} \]
\[ [A] \quad Y \leftarrow Z_1 \]