1 Problems

2 Solutions
1. Write a program fragment that exchanges the value of two variables. In other words, if $Z_1 = a$ and $Z_2 = b$, after executing this fragment we have $Z_1 = b$ and $Z_2 = a$.

2. Write a function in $S$ that computes the remainder of the division of $m$ by $n$.

3. Let $gcd(x_1, x_2)$ be the greatest common divisor of $x_1$ and $x_2$. Write a program in $S$ that computes $f$.

4. Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be the function defined by

$$f(x) = \begin{cases} 
1 & \text{if } x \text{ is even}, \\
0 & \text{if } x \text{ is odd}.
\end{cases}$$

Write a program in $S$ that computes $f$.

5. Let $f$ be a partial function such that $f(x) = 1$ if $x$ is even, and $f(x) \uparrow$ if $x$ is odd. Write a program in $S$ that computes $f$. 

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Problem 1: Write a program fragment that exchanges the value of two variables. In other words, if \( Z_1 = a \) and \( Z_2 = b \), after executing this fragment we have \( Z_1 = b \) and \( Z_2 = a \). The following program fragment \( Q(Z_1, Z_2) \) solves the problem:

\[
\begin{align*}
Z_2 & \leftarrow Z_1 + Z_2 \\
Z_1 & \leftarrow Z_2 - Z_1 \\
Z_2 & \leftarrow Z_2 - Z_1
\end{align*}
\]

An example of the sequence of states of this fragment:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>initial state</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>after executing ( Z_2 \leftarrow Z_1 + Z_2 )</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>after executing ( Z_1 \leftarrow Z_2 - Z_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>after executing ( Z_2 \leftarrow Z_2 - Z_1 ).</td>
</tr>
</tbody>
</table>
Problem 2: Write a function in $S$ that computes the remainder of the division of $m$ by $n$.
Examples: $f(20, 7) = 6$, $f(7, 20) = 7(7 = 20 \times 0 + 7)$.

\[
[B] \quad \text{IF } (X_1 \leq X_2) \text{ GOTO } A
\]
\[
X_1 \leftarrow X_1 - X_2
\]
\[
\text{GOTO } B
\]

\[
[A] \quad Y \leftarrow X_1
\]
\[
\text{GOTO } E
\]

An example of the sequence of states of this fragment:

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>initial state</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Problem 3: The greatest common divisor $gcd(m, n)$ can be computed as follows in Python in a non-recursive manner:

```
>>> def gcd(m,n):
    while n > 0:
        m = m % n
        p = m
        m = n
        n = p
    return m
```
\[ Z_1 \leftarrow X_1 \]
\[ Z_2 \leftarrow X_2 \]

[B] IF \( Z_2 = 0 \) GOTO A
\[ Z_1 \leftarrow f(Z_1, Z_2) \]
\[ Q(Z_1, Z_2) \]
GOTO B

[A] \( Y \leftarrow Z_1 \)
Problem 4: Let $f : \mathbb{N} \to \mathbb{N}$ be the function defined by

$$f(x) = \begin{cases} 
1 & \text{if } x \text{ is even,} \\
0 & \text{if } x \text{ is odd.}
\end{cases}$$

Write a program in $S$ that computes $f$.
Solution:

IF $X = 0$ GOTO $B$
IF $X = 1$ GOTO $E$

[A] $X \rightarrow X - 2$
IF $X > 2$ GOTO $A$
IF $X = 0$ GOTO $B$
$Y \leftarrow 0$
GOTO $E$

[B] $Y \leftarrow 1$
Problem 5: Let $f$ be a partial function such that $f(x) = 1$ if $x$ is even, and $f(x) \uparrow$ if $x$ is odd. Write a program in $S$ that computes $f$.

Solution:

IF $X = 0$ GOTO B
IF $X = 1$ GOTO C

[A] $X \rightarrow X - 2$
    IF $X > 2$ GOTO A
    IF $X = 0$ GOTO B

[C] GOTO C

[B] $Y \leftarrow 1$