Homework 1
Due Monday October 7, 2013

Please print your name here:

1. Using macros write a program in \( S \) that computes the function \( f(x) = 2x \) for \( x \in \mathbb{N} \); write the same program without using macros.

2. Write a program that computes the greatest common divisor of \( x_1 \) and \( x_2 \), \( \text{gcd}(x_1, x_2) \). You may use macros.

3. Let \( \mathcal{P} \) be the program

   \[
   \begin{align*}
   Y & \leftarrow X \\
   [A] & \text{IF } X_2 = 0 \text{ GOTO E} \\
   & Y \leftarrow Y + 1 \\
   & Y \leftarrow Y + 1 \\
   & X_2 \leftarrow X_2 - 1 \\
   & \text{GOTO A}
   \end{align*}
   \]

   What is \( \psi^{(1)}_{\mathcal{P}}(r_1), \psi^{(2)}_{\mathcal{P}}(r_1, r_2), \) and \( \psi^{(3)}_{\mathcal{P}}(r_1, r_2, r_3) \)?

4. A straightline program in \( S \) is a program that contains no instruction of the form

   \[
   \text{IF } V \neq 0 \text{ GOTO L.}
   \]

   Show by induction on the length of the program that if the length of a straightline program \( \mathcal{P} \) is \( k \), then \( \psi^{(1)}_{\mathcal{P}}(x) \leq k \) for every \( x \in \mathbb{N} \).

5. Let \( P(x) \) be a computable predicate. Show that the function \( f \) defined by

   \[
   f(x_1, x_2) = \begin{cases} 
   x_1x_2 & \text{if } P(x_1 + x_2), \\
   \uparrow & \text{otherwise}
   \end{cases}
   \]

   is partially computable.