Homework 2

*Posted: October 3, 2018*  
*Due: October 17, 2018*

1. Let $A = \{0, 1\}$ be an alphabet that consists of two binary digits. Denote by $f(x)$ the numerical equivalent of $x$, as we did in class. Design a DFA that accepts the set of words $\{x \in \{0, 1\}^* \mid f(x) \text{ is a multiple of 6}\}$.

2. Construct deterministic finite automata that accept the following languages over the alphabet $A = \{a, b, c\}$:
   
   (a) The set of all words that begin with $ab$ and end with $ba$.
   
   (b) The set $\{bab\}$.
   
   (c) The set $A^* - \{bab\}$.

3. Construct non-deterministic finite automata that accept the following languages over the alphabet $A = \{a, b, c\}$:
   
   (a) The set of all words that begin with $ab$ and end with $ba$.
   
   (b) The set $\{bab\}$.
   
   (c) The set $A^* - \{bab\}$.

4. Prove or disprove the following statements. Proving requires an argument; disproving requires a counterexample.
   
   (a) Every language is contained in a regular language.
   
   (b) Every nonempty language contains a nonempty regular language.
   
   (c) The union of a collection of regular languages is a regular language.
   
   (d) If $L_0, L_1$ are regular languages and $L_0 \subseteq L \subseteq L_1$, then $L$ is a regular language.
5. Let $A$ be an alphabet and let $a \in A$ be a symbol. If $k$ is a natural number, construct a nondeterministic finite automaton that accepts the language $L_{k,a} = \{uav \mid u, v \in A^* \text{ and } |v| = k\}$. 