Homework 5

Posted: November 24, 2018
Due: December 10, 2018

1. Prove that the language

$$L = \{a^n b^m \mid m = n \text{ or } m = 2n\}$$

   is context-free.

2. Prove that if G is a context-free grammar in Chomsky normal form, then for any $x \in L(G)$ with $|x| = n$ the derivation of x requires $2n - 1$ steps.

3. Let $G = (A_N, A_T, S, P)$ be a context-free grammar in Chomsky normal form. Prove that if there exists a word $w \in L(G)$ generated by a derivation that uses more than $2|P| - 1$ steps, then $L(G)$ is infinite.

4. Prove that:

   - the language $\{xyx \mid x, y \in \{a, b\}^*\}$ is not context-free;
   - the language $\{xyx^R \mid x, y \in \{a, b\}^*\}$ is context free.

5. Prove that the language $\{a^{2^n} \mid n \in \mathbb{N}\}$ is not context-free.