1. Let \( A = \{0, 1\} \) be an alphabet that consists of two binary digits. Denote by \( f(x) \) the numerical equivalent of \( x \), as we did in class. Design a dfa that accepts the set of words \( \{x \in \{0, 1\}^* \mid f(x) \text{ is a multiple of } 6\} \).

2. Construct deterministic finite automata that accept the following languages over the alphabet \( A = \{a, b, c\} \):

   (a) The set of all words that begin with \( ab \) and end with \( ba \).
   (b) The set \( \{bab\} \).
   (c) The set \( A^* - \{bab\} \).

   Justify your solutions.

3. Construct non-deterministic finite automata that accept the following languages over the alphabet \( A = \{a, b, c\} \):

   (a) The set of all words that begin with \( ab \) and end with \( ba \).
   (b) The set \( \{bab\} \).
   (c) The set \( A^* - \{bab\} \).

   Justify your solutions.

4. Prove or disprove the following statements. Proving requires an argument; disproving requires a counterexample.

   (a) Every language is contained in a regular language.
   (b) Every nonempty language contains a nonempty regular language.
(c) The union of a collection of regular languages is a regular language.

(d) If $L_0, L_1$ are regular languages and $L_0 \subseteq L \subseteq L_1$, then $L$ is a regular language.

5. Let $A$ be an alphabet and let $a \in A$ be a symbol. If $k$ is a natural number, construct a nondeterministic finite automaton that accepts the language $L_{k,a} = \{uav \mid u, v \in A^* \text{ and } |v| = k\}$. 