1. Construct deterministic finite automata that accept the following languages over the alphabet $A = \{a, b, c\}$:

   (a) The set of all words that begin with $ab$ and end with $ba$.
   
   (b) The set $\{bab\}$.
   
   (c) The set $A^* - \{bab\}$.
   
   (d) The set of all words $x \in A^*$ that contain at least three $a$s.

2. Draw a transition diagram for a nondeterministic finite automaton $\mathcal{M}$ that accepts all strings over the alphabet $A = \{0, 1\}$ that begin in 10 and end in 11. Construct the dfa that accepts the same language as $\mathcal{M}$.

3. Let $A$ be an alphabet and let $a \in A$ be a symbol. If $k$ is a natural number, construct a nondeterministic finite automaton that accepts the language $L_{k,a} = \{uav \mid u, v \in A^* \text{ and } |v| = k\}$.

4. Construct nondeterministic finite automata that accept the following languages over $A = \{a, b\}$:

   (a) The set of words that begin with an $a$ or begin and end with a $b$.
   
   (b) The set $\{a\}^* \cup \{b\}^*$.
   
   (c) The set of words that contain $bab$ as an infix.
5. Let $\mathcal{M} = (A, Q, \delta, q_0, F)$ be a ndfa. Define $\phi(q, w, q')$ as the number of paths in the graph of $\mathcal{M}$ from the state $q$ to the state $q'$ with label $w$. Prove that for the automaton $\mathcal{M} = (\{a, b\}, \{q_0, q_1\}, \delta, q_0, \{q_0\})$ whose graph is given in Figure 1 we have $\phi(q_0, a^n, q_0) = f_{n+1}$, where $f_n$ is the $n^{\text{th}}$ Fibonacci number and $n \geq 0$. 

Figure 1: Graph of the Automaton $\mathcal{M}$