CS671 - Machine Learning
Homework 3
Posted April 20, 2015
Due May 1, 2015

1. Determine the interior, closure, and border of the following sets:

   (a) $S_1 = \{\mathbf{x} \in \mathbb{R}^2 \mid x_1^2 + x_2^2 \leqslant 1 \text{ and } x_2 > 1\}$;

   (b) $S_2 = \{\mathbf{x} \in \mathbb{R}^2 \mid x_1^2 + x_2^2 \leqslant 1 \text{ and } x_1 + x_2 = 1.4\}$;

   (c) $S_3 = \{\mathbf{x} \in \mathbb{R}^2 \mid x_1^2 + x_2^2 \leqslant 1 \text{ and } x_1 + x_2 = 1.5\}$.

2. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be the function

$$f(\mathbf{x}) = x_1^2 + 4x_2^2$$

   and let S be the set of feasible solutions defined by the restrictions

$$\begin{aligned} 5x_1 + 8x_2 - 41 &\leqslant 0, \\ x_1 &\geqslant 0, \\ x_2 &\geqslant 0. \end{aligned}$$

   Using Fritz John theorem determine the point of minimum for f.

3. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be the function defined by

$$f(\mathbf{x}) = x_1^2 + 2x_1x_2 - 10x_1 + 5x_2.$$

   Is f convex, concave, or neither? Why?

4. Let $f_1 : \mathbb{R}^n \longrightarrow \mathbb{R}$ and $f_2 : \mathbb{R}^n \longrightarrow \mathbb{R}$ be two convex functions. Prove that the function f defined by $f(\mathbf{x}) = \max\{f_1(\mathbf{x}), f_2(\mathbf{x})\}$ is convex.

5. Consider the following problem

$$\begin{aligned} &\text{minimize } x_1^2 + 2x_2^2 \\ &\qquad\text{subject to } x_1 + x_2 - 1 \leqslant 0. \end{aligned}$$

   Find a point satisfying the Kuhn-Tucker conditions. Is that point an optimal solution?