1. Let \( f(x) \) be the greatest number \( n \) such that \( n^2 < x \). Write a program in \( S \) that computes \( f \).

2. Let \( P \) be the program

   \[
   Y \leftarrow X_1 \\
   [A] \quad \text{IF } X_2 = 0 \text{ GOTO } E \\
   Y \leftarrow Y + 1 \\
   Y \leftarrow Y + 1 \\
   X_2 \leftarrow X_2 - 1 \\
   \text{GOTO } A
   \]

   What is \( \Psi_P^{(1)}(r_1) \)? \( \Psi_P^{(2)}(r_1, r_2) \)? \( \Psi_P^{(3)}(r_1, r_2, r_3) \)?

3. A unary function \( f(x) \) is said to be partially \( n \)-computable if it is computed by some \( S \) program \( P \) such that \( P \) has no more than \( n \) instructions, every variable in \( P \) is among \( X, Y, Z_1, \ldots, Z_n \) and every label in \( P \) is among \( A_1, \ldots, A_n, E \). Prove that if a unary function \( f : \mathbb{N} \rightarrow \mathbb{N} \) is computed by a program with no more than \( n \) instruction, then \( f \) is partially \( n \)-computable.

4. Let \( P(x) \) be a computable predicate. Show that the function \( f \) defined as

   \[
   f(x_1, x_2) = \begin{cases} 
   x_1 + X_2 & \text{if } P(x_1 + x_2) \\
   \uparrow & \text{otherwise}
   \end{cases}
   \]

   is partially computable.
5. Let $f(x)$ be a partially computable but not total function, let $M$ be a finite set of numbers such that $f(m) \uparrow$ for all $m \in M$ and let $g$ be an arbitrary partially computable function. Show that $h(x)$ defined as

$$h(x) = \begin{cases} g(x) & \text{if } x \in M, \\ f(x) & \text{otherwise} \end{cases}$$

is partially computable.