Homework 3

Posted: October 21, 2020  Note that on Wednesday, Nov. 11 we do not have a class, due to a legal holiday; however, the homework is due at 7:00pm on that day or earlier.

1. Find the program $P$ such that $\#(P) = 575$.

2. Let $\text{HALT}^1(x)$ be the predicate defined as $\text{HALT}^1 = \text{HALT}(\ell(x), r(x))$. Show that $\text{HALT}^1$ is not computable.

3. Let $f(x_1, \ldots, x_n)$ be computed by program $P$ where $\#(P) = p$ and suppose that for some primitive recursive function $g$,

   $\text{STP}^{(n)}(x_1, \ldots, x_n, p, g(x_1, \ldots, x_n))$

   is TRUE for all $x_1, \ldots, x_n$. Show that $f(x_1, \ldots, x_n)$ is primitive recursive.

4. Let $B = \{ f(n) \mid n \in \mathbb{N} \}$, where $f$ is a strictly increasing function (that is, $f(n + 1) > f(n)$ for all $n$). Prove that $B$ is recursive, that is, $P_B$ is computable.

5. Show that there is no computable function $f(x)$ such that $f(x) = \Phi(x, x) + 1$ whenever $\Phi(x, x)$ is defined.