Homework 3

Posted: October 21, 2020  
Due: November 5, 2020

1. Find the program $\mathcal{P}$ such that $\#(\mathcal{P}) = 575$.

2. Let $\text{HALT}^1(x)$ be the predicate defined as $\text{HALT}^1 = \text{HALT}(\ell(x), r(x))$. Show that $\text{HALT}^1$ is not computable.

3. Let $f(x_1, \ldots, x_n)$ be computed by program $\mathcal{P}$ where $\#(\mathcal{P}) = p$ and suppose that for some primitive recursive function,

$$\text{STP}^{(n)}(x_1, \ldots, x_n, p, g(x_1, \ldots, g(x_n)))$$

is TRUE for all $x_1, \ldots, x_n$. Show that $f(x_1, \ldots, x_n)$ is primitive recursive.

4. Let $B = \{f(n) \mid n \in \mathbb{N}\}$, where $f$ is a strictly increasing function (that is, $f(n + 1) > f(n)$ for all $n$). Prove that $B$ is recursive, that is, $P_B$ is computable.

5. Show that there is no computable function $f(x)$ such that $f(x) = \Phi(x, x) + 1$ whenever $\Phi(x, x)$ is defined.