CS/Math 320 Discrete Mathematics Fall 2017

Solution to Test 1

Problem 1. Determine whether each of these conditional statements is true or false.

a) If 1+1=2, then 2+2=5

Solution: FALSE

b) If 1+1=3, then 2+2=4

Solution: TRUE

c) If 1+1=3, then 2+2=5

Solution: TRUE

d) If monkeys can fly, then 1+1=3

Solution: TRUE

Problem 2. Construct the truth table for the following compound proposition: $(p \to q) \leftrightarrow (\neg q \to \neg p)$

Solution:

p	q	p->q	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	T
T	T	T	T	T

Problem 3. Show that the argument form below is valid

Premises:

$p \wedge t \to r \vee s$ (1),

$q \to u \wedge t$ (2),

$u \to p$ (3),

$\neg s$ (4)

Conclusion: $q \to r$

Solution:

We have:

u->p (due to 3)

———

∴ u ∧ t → p ∧ t

because

- if t=T, u ∧ t → p ∧ t = u->p = T
- else, u ∧ t → p ∧ t = F->F = T

either way, u ∧ t → p ∧ t is T.

u ∧ t → p ∧ t

p ∧ t → r ∨ s (due to 1)

———

∴ u ∧ t → r ∨ s

q → u ∧ t (due to 2)

———

∴ q → r ∨ s

¬s (due to 4)

———

∴ q → r

because with s=F, we have r ∨ s = r.

Problem 4. Show in at least two different ways that the compound propositions ¬p ∨ (r →¬q) and ¬p ∨ ¬q ∨ ¬r are equivalent.

Solution:

One is using the TRUTH table

One is using the rule: a->b ≡ ¬a ∨ b

¬p ∨ (r →¬q) ≡ ¬p ∨ (¬r ∨ ¬q) ≡ ¬p ∨ ¬q ∨ ¬r

Problem 5. Find a compound proposition involving the propositional variables p, q, r, and s that is true when exactly three of these propositional variables are true and is false otherwise.

Solution:

$A = (p \wedge q \wedge r \wedge \neg s) \vee (p \wedge q \wedge s \wedge \neg r) \vee (q \wedge r \wedge s \wedge \neg p) \vee (q \wedge r \wedge s \wedge \neg q)$

Problem 6. Use existential and universal quantifiers to express the statement "No one has more than three grandmothers" using the propositional function G(x , y), which represents "x is the grandmother of y."

Solution:

"No one has more than three grandmothers"

is equivalent to

"Everyone has at most three grandmothers"

$\forall x\ \forall y\ \forall z\ \forall u\ \forall v\ G(y, x) \wedge G(z, x) \wedge G(u, x) \wedge G(v, x) \rightarrow (y=z) \vee (y=u) \vee (y=v) \vee (z=u) \vee (z=v) \vee (v=u)$

(if there exist four grandmothers y, z, u, v of x, at least two of them must be the same person)